In recent years, there has been growing interest at local, state, and national levels in establishing a firm research basis for the instructional materials and programs that are used in classrooms. This paper describes research that supports the instructional design and other specific features of Problem-Solving Experiences: Making Sense of Mathematics (PSE).

PSE is a revision and enhancement of the previous problem-centered program written by Dr. Randall Charles, Dr. Frank Lester Jr., and others, Problem-Solving Experiences in Mathematics (published in 1985). The instructional model for these programs evolved from three principles for teaching problem solving that accrued from research (Kantowski 1981; Stengel et al. 1977) and from master teachers:

(a) It is necessary to solve problems in order to improve one’s problem-solving ability.

(b) Problem-solving ability develops slowly over a prolonged period of time.

(c) Most students benefit greatly from systematically planned problem-solving instruction.

The important elements of the PSE instructional model are as follows:

1. A focus on the key phases of problem solving: Read and Understand, Plan and Solve, Look Back and Check

2. An emphasis on extensive experience with solving nonroutine (or process) problems

3. An emphasis on the development of students’ abilities to select and use a variety of problem-solving strategies

4. Use of a specific teaching strategy for problem solving

Primary Research Conducted by Dr. Charles and Dr. Lester

During 1982–1983, Dr. Charles and Dr. Lester conducted a major research study on the effectiveness of the instructional model used for the original program and replicated in the new PSE. That research report was published in the Journal for Research in Mathematics Education (1984). Overall, Charles and

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Lester found that children can learn how and when to use problem-solving strategies to successfully solve problems when provided with explicit instruction on the strategies. In particular, treatment or the experimental classes scored significantly higher than the control classes on measures of ability to understand problems, plan solution strategies, and get correct results.

**Purposes of the Study**

One purpose of the study was to compare the problem-solving performance of students who participated in the Mathematical Problem Solving (MPS) program to that of students whose only exposure to problem solving was that provided by the regular textbook. (MPS was the name given to the program during the study.) A second purpose was to examine the nature of the changes in students’ problem-solving performance while participating in MPS. A third purpose was to investigate teachers’ attitudes toward and confidence in teaching problem solving using the MPS materials and guidelines. Information was also sought regarding teachers’ views about particular aspects of the program (its content, organization, and effectiveness with low achievers), the teachability of the program, the evidence they saw of transfer of skills and processes learned, and changes in students’ affective behavior.

**Subjects**

Thirty-six schools in four counties in West Virginia with similar levels of achievement on the Comprehensive Test of Basic Skills (CTBS, 1973) were identified, and 23 fifth-grade and 23 seventh-grade teachers were asked to participate in a problem-solving project. Complete sets of data were obtained from 451 students at Grade 5 and 485 students at Grade 7. The original numbers of students were 8% and 10% larger for each grade, respectively. Individual students’ scores on the CTBS were not available, but achievement data for the participating schools were at approximately the sixty-fifth percentile for mathematics computation and the fifty-eighth percentile for mathematics applications. Neither the teachers nor the students in the study had had any prior special training related to problem solving.

**Procedures**

Twelve treatment and 11 control classes were selected at Grade 5, and 10 treatment and 13 control classes were selected at Grade 7. (Two of the 12 Grade 5 treatment classes were later dropped from the study because of large amounts of missing data.) The teachers’ classes were assigned to treatment or control groups so as to maintain roughly equal mean achievement on the CTBS. No school had both a treatment and a control class. The teachers of the control classes were told that they were participating in a problem-solving project and that they would be helping assess the difficulty of new problem-solving material by having their students solve some problems throughout the school year. The teachers of the treatment classes received 3 hours of training on the use of the program prior to the study.

The study lasted 23 weeks. A pretest was administered at the beginning to all classes, and a posttest was administered at the end to all classes. Students in the treatment classes also received “intermediate assessments,” one after 8 weeks and another after 16 weeks. All tests were made up of two problem types: multiple-step problems and nonroutine problems.

Each treatment teacher was interviewed at the end of the study. Students’ problem-solving performance was assessed by an analysis of their written performance using a holistic assessment instrument. The scoring scheme gave scores on three dimensions: understanding of the problem, planning to solve the problem, and the result of the work.

**Major Finding**

Table 1 shows that for both grades, the differences in the pretest scores and posttest scores were statistically significant, favoring the users of the program in all areas except the results for multiple-step problems at Grade 5. Note that the pretest score at Grade 5 on the multiple-step problems for the treatment students was lower than that for the control students, but on the posttest, the score for the treatment students was higher than that for the control students. The difference, however, was not statistically significant.
Other Findings

Some of the other findings from the study are listed below.

1. MPS improves students’ abilities to understand problems and to plan solution strategies faster than it improves their abilities to get correct results.

2. A significant factor in forming teachers’ attitudes toward a problem-solving program is the program’s structure. MPS provides a set of specific guidelines for teachers’ actions that seem to have a very positive influence on their attitudes.

3. Among the most important benefits of MPS are the following:
   (a) The program improves students’ willingness to engage in problem solving.
   (b) Students gain confidence in their ability to succeed in problem solving.
   (c) Students learn “how to think.”

4. Fundamental to an evaluation of any instructional program is a consideration of the extent to which the program requires specialized expertise or material to be implemented successfully. A particular strength of MPS is that it is organized in a way that classroom teachers can use it with very little in-service training.

Research Drawn From Additional Sources

In addition to conducting the previously cited primary research, the authors conducted a systematic analysis of the body of research on teaching mathematics through problem solving. As a foundation, they aligned the program with the Standards established by the National Council of Teachers of Mathematics (NCTM). According to the Standards, “Problem solving is an integral part of all mathematics learning.” The NCTM Problem-Solving Standard is described below. Research supporting the Principles and Standards for School Mathematics is documented in a separate book, A Research Companion to Principles and Standards for School Mathematics, published by NCTM (2003).

Problem-Solving Standard

“Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.”

—NCTM Principles and Standards for School Mathematics, p. 52

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In developing the instructional design of this program, the authors applied research findings from the following specific areas:

- Consistent Lesson Structure
- Continual Practice and Review
- Problem-Solving Strategies
- Representations
- Algebraic Thinking
- Estimation
- Journal Writing

**Consistent Lesson Structure**

Jitendra et al. (1999), in examining the instructional design of seven mathematics programs, found that all learners, but particularly special-needs learners, are aided when lessons are built around a consistent structure.4

PSE is built on a design of structured instruction consisting of a carefully crafted sequence of the following five problem types in each of the 30 problem sets per grade level: **Strategy**, **Estimation**, **One-step (Algebraic Thinking at Grades 6–8)**, **Multiple-step**, and **Concept**. Throughout the program, students are aided by the use of the following consistent problem-solving process: **Read and Understand**, **Plan and Solve**, and **Look Back and Check**. This problem-solving process is based on the work of George Polya (1957).5

**Continual Practice and Review**

A number of studies have shown that continual practice and review promotes student achievement at all grade levels. Good and Grouws (1979) found that fourth-grade students who were exposed to a planned program of review performed significantly better than students in a control group without a program of regular review.6

Applying this research, the authors concluded that students must solve many problems on a regular basis in order to improve their problem-solving ability. At each grade level, PSE provides 30 sets of 5 problem-solving experiences each—enough material for nearly every day of the entire school year.

Related to continual practice and review is the issue of performance on basic skills. In any good instructional program, students are expected to master basic skills and learn algorithms in addition to engaging in explorations of worthwhile problems. However, some people—parents and teachers alike—worry that the development of students’ higher-order thinking skills in teaching through problem solving may come at the expense of the development of basic mathematical skills. Cai (2003) analyzed several studies involving elementary and middle school classrooms that have addressed this legitimate question directly. These studies reveal that in comparison to students in control groups, students experiencing problem-based instruction usually have higher levels of mathematical understanding and problem-solving skills—and have at least comparable basic numerical skills.7

With respect to the mathematics content in PSE, it is important to note that key mathematics concepts and skills are embedded in the problem sets. The wrap-around Teaching Notes in the Teacher Edition for each Student Book page indicate the skill, concept, or strategy that is addressed in each problem on the page. For a correlated list of the math concepts and skills that underlie the problems in the Student Book for this grade level, see page xi in the Teacher Edition.

**Instruction on Problem-Solving Strategies**

In addition to the Charles and Lester research previously cited, other studies strongly concur that there should be direct instruction on the teaching of problem-solving strategies. Suydam (1980) found that

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problem-solving performance is enhanced by teaching students to use a variety of strategies. Hembree and Marsh (1993) found that problem-solving practice without direct instruction on strategies did not produce improvement. They found that developing skill with diagrams gave the most pronounced effects on problem-solving performance (at all grades), followed by training in the translation of words into symbols (beginning at Grade 4).\(^9\)

In PSE, the first problem in every set is a strategy problem. A list of the strategies that are taught and applied in the program is shown below. The two key strategies cited by Hembree and Marsh are emphasized in the program (Draw a Picture; Write a Number Sentence/Write an Equation).

### Strategies Developed in Problem-Solving Experiences
- Act It Out/Use Objects
- Make a Table
- Look for a Pattern
- Use/Made a Graph
- Make a Simpler Problem
- Write a Number Sentence/Write an Equation
- Draw a Picture
- Try, Check, and Revise
- Make an Organized List
- Use Logical Reasoning
- Work Backward

### Representations
One of the most significant changes from the 1989 NCTM Curriculum and Evaluation Standards for School Mathematics to the 2000 NCTM Principles and Standards for School Mathematics was the inclusion of the Representation Standard (see below). The term representation refers both to process and to product. As a process it refers to the act of creating in one's mind one's own mental image or images of a mathematical idea. As a product it refers to some physical form of that idea, such as a picture, a diagram, a graph, or a manipulative.

#### Representation Standard
“Instructional programs from prekindergarten through grade 12 should enable all students to—
- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.”

—NCTM Principles and Standards for School Mathematics, p. 67

Why is the idea of representation so important? Simply stated, the more ways a student can think about a mathematical concept, the better that student will understand the underlying mathematical idea. A number of research studies support the effectiveness of using representations to develop concepts and to help solve problems.

- In a meta-analysis of 60 research studies, Sowell (1989) found that for students of all ages, mathematics achievement is increased and students’ attitudes toward mathematics are improved with the long-term use of manipulative materials.\(^10\)
- In a study of more than 7,000 students, Wenglinsky (2000) found that students whose teachers conduct hands-on learning activities outperform their peers

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by more than 70% of a grade level in math on the National Assessment of Educational Progress (NAEP).\textsuperscript{11}

- According to Nickerson (1994), students who use and understand diagrams are better able to handle abstract concepts because they have a concrete physical model to work from.\textsuperscript{12}

In PSE, two problem-solving strategies (Act It Out/Use Objects and Draw a Picture) specifically utilize representations as a way to help students solve problems. These strategies utilize physical models or diagrams to represent a problem and to provide concrete and/or visual clues.

In PSE, the use of diagrams is not confined to the two aforementioned types of strategy lessons. Rather, it is embedded throughout the problem-solving experiences. Diagrams are given particular emphasis in the development of one-step problems. Such diagrams show how the quantities in a problem are related to each other and why a particular operation or operations can be used to solve the problem. The consistent use of diagrams to show operation meanings not only improves students’ abilities to solve one-step word problems, but it also provides a solid foundation for algebra. Ferrucci et al (2003) found, in part from their observations of Singapore schools and curricula, that modeling is a powerful tool for students to use to enhance their problem-solving and algebraic reasoning skills. They found that students are thus able to use representations to solve problems involving algebraic thinking before they take a formal course in algebra.\textsuperscript{13}

Algebraic Thinking

The NCTM \textit{Principles and Standards for School Mathematics} identifies algebra as a strand in the curriculum from Grades pre-K through 12 (see below). According to NCTM, “All students should learn algebra.”\textsuperscript{14} In light of the growing importance placed on algebra in the curriculum, the authors of the new PSE have included an \textit{Algebraic Thinking} problem in each of the problem sets at Grades 6–8. As mentioned earlier, the representations used for one-step problems are important elements in algebraic thinking. In addition, the \textit{Algebraic Thinking} problems in PSE develop concepts such as patterns, interpreting graphs, interpreting equation language, and more. This work with algebra is in addition to the components of algebraic thinking (such as writing a number sentence) that were already embedded in the original program at all grades.

\textbf{Algebra Standard}

“Instructional programs from prekindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.”

—NCTM \textit{Principles and Standards for School Mathematics}, p. 37

\textbf{Estimation}

Traditionally, estimation has been thought of as a supplemental skill. However, based on surveys of adults, Carlton (1980) found that most of the mathematics used in everyday living relies far more on mental computation and estimation than on traditional computation.\textsuperscript{15} According to Reys (1988), “Research has documented that teaching estimation

\begin{itemize}
  \item \textsuperscript{13} Beverly J. Ferrucci, Ban-Har Yeap, and Jack A. Carter, “A Modeling Approach for Enhancing Problem Solving in the Middle Grades.” \textit{Mathematics Teaching in the Middle School}, 8, no. 9 (2003), pp. 470–475.
\end{itemize}
strategies and related skills on a systematic basis over a year-long period can produce dramatic improvements in estimation performance.”

“Estimation relates to every important mathematics concept and skill developed in elementary school.”

In PSE, the second problem in every set is an estimation problem. In addition, the use of estimation and estimation strategies (such as rounding, use of compatible numbers, and so on) is embedded in the problem-solving process throughout the program. In particular, students use estimation to determine if their answer is reasonable (in the Look Back and Check problem-solving phase).

Journal Writing
A number of studies support the use of journal writing in the mathematics classroom.

- In a study of fifth and sixth graders, DiPillo et al. (1997) found that the use of journals enabled teachers to gain insights into students’ conceptual and procedural knowledge of, and attitudes and feelings about, mathematics. Students’ responses suggested that they found journals to be “a way of sharing their thoughts and feelings about mathematics, a learning tool that facilitated retention, and an effective way to communicate with their teachers.”
- Jurdak and Abu Zein (1998) found that journal writing in mathematics provides cognitive benefits to students in terms of increased procedural knowledge, conceptual understanding, and mathematical communication.

- In analyzing several studies, Mason and McFeetors (2002) found that the use of writing in a mathematics class provides many benefits. It develops students’ understanding of ideas, enables teachers to identify incomplete conceptions, and contributes to a more interactive relationship between students and teacher.

In PSE, every page of the Student Book includes one Math Journal question. These open-ended questions provide students with ample opportunities to explain their reasoning and to justify their thought processes and strategies. Such writing encourages students to show their work while promoting mathematical communication—an essential part of mathematics and mathematics education. Communication is one of the NCTM Standards (see below).

Communication Standard
“Instructional programs from prekindergarten through grade 12 should enable all students to—
- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.”
—NCTM Principles and Standards for School Mathematics, p. 60

Attitudes and Beliefs
Problem-solving instruction can promote the development of helpful dispositions, beliefs, and attitudes related to problem solving in particular and to mathematics in general. Beliefs, such as “most
problems can be solved in more than one way” and “many problems have more than one answer” are helpful when solving problems. Dispositions such as not “giving up” or resisting “stopping before checking one’s work” can be developed through problem solving. In his research synthesis, McLeod (1989) concluded that there is “a wealth of data to document the ways in which beliefs, attitudes, and emotions are related to the performance of both students and teachers in mathematics classrooms.”

“Mathematically proficient people believe that mathematics should make sense, that they can figure it out, that they can solve mathematical problems by working hard on them, and that becoming mathematically proficient is worth the effort.”


An important goal of Problem Solving Experiences: Making Sense of Mathematics is to promote positive attitudes and helpful beliefs about doing mathematics. In particular, the student pages in PSE frequently provide user-friendly “Think” and “Remember” hints to offer practical suggestions designed to promote student perseverance and success.

Conclusion

The primary research conducted by Dr. Charles and Dr. Lester supports the conclusion that the key elements of the instructional model in Problem Solving Experiences: Making Sense of Mathematics lead to a significant improvement in students’ problem-solving performance. Also, teachers view the organization of the program positively and find it easy to use. Furthermore, there is a firm research basis for other elements of the program, including consistent lesson structure, continual practice and review, the development of algebraic reasoning, the development of estimation methods, the emphasis on representations, and the use of journal writing.