Metacognition and Mathematical Problem Solving in Grade 3

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Abstract

This article presents an overview of two studies that examined the relationship between metacognition and mathematical problem solving in 165 children with average intelligence in Grade 3 in order to help teachers and therapists gain a better understanding of contributors to successful mathematical performance. Principal components analysis on metacognition revealed that three metacognitive components (global metacognition, off-line metacognition, and attribution to effort) explained 66% to 67% of the common variance. The findings from these studies support the use of the assessment of off-line metacognition (essentially prediction and evaluation) to differentiate between average and above-average mathematical problem solvers and between students with a severe or moderate specific mathematics learning disability.

Flavell introduced the concept of metacognition in 1976, in the context of developmental psychology and research on metamemory (Simons, 1996). He defined metacognition as “one’s knowledge concerning one’s own cognitive processes and products or anything related to them…. Metacognition refers furthermore to the active monitoring of these processes in relation to the cognitive objects or data on which they bear, usually in service of some concrete goal or objective” (Flavell, 1976, p. 232).

To gain a better understanding of successful mathematical performance, metacognition seems to be important (Lucangeli & Cornoldi, 1997). Nowadays, metacognition has become a general multidimensional and overarching construct (Boekaerts, 1999), enabling learners to adjust accordingly to varying problem-solving tasks, demands, and contexts (Allen & Armour-Thomas, 1992; Montague, 1998). Simons (1996) postulated a difference between metacognitive knowledge, executive control (or metacognitive skills), and metacognitive conceptions (or metacognitive beliefs).

Metacognitive knowledge has been described as the knowledge and the deeper understanding of cognitive processes and products (Flavell, 1976). In mathematics, for example, children may know that they have to check themselves in multidigit divisions but not while solving one-digit additions. Three components of metacognitive knowledge have been described. Declarative metacognitive knowledge was found to be “what is known in a propositional manner” (Jacobs & Paris, 1987, p. 259) or the assertions about the world and the knowledge of the influencing factors (memory, attention, etc.) of human thinking. Procedural metacognitive knowledge can be described as “the awareness of processes of thinking” (Jacobs & Paris, 1987, p. 259) or the knowledge of the methods for achieving goals and the knowledge of how skills work and how they are to be applied. Procedural knowledge is necessary, according to Montague (1992), to apply declarative knowledge efficaciously and to coordinate multiple cognitive and metacognitive problem solving. Conditional or strategic metacognitive knowledge is considered to be “the awareness of the conditions that influence learning such as why strategies are effective, when they should be applied, and when they are appropriate” (Jacobs & Paris, 1987, p. 259). Conditional knowledge, according to Montague (1992), enables a learner to select appropriate strategies and to adjust behavior to changing task demands. These metacognitive components may therefore help children to know how to study a new timetable (procedural knowledge), to make use of the awareness of previously studied number facts (declarative knowledge), and to select appropriate study behavior (conditional knowledge).

According to Brown (1980), executive control or metacognitive skills can be seen as the voluntary control people have over their own cognitive processes. A substantial amount of data has been accumulated on four metacognitive skills: prediction, planning, monitoring, and evaluation (e.g., Lucangeli & Cornoldi, 1997). In mathematics, prediction refers to activities aimed at differentiating difficult exercises (e.g., $126 \div 5 = \ldots$) from the easy ones (e.g., $126 - 5 = \ldots$) in order to be able to concentrate on and persist more
in the high-effort tasks. Planning involves analyzing exercises (e.g., "It is a division exercise in a number problem format"), retrieving relevant domain-specific knowledge and skills (e.g., how to do divisions), and sequencing problem-solving strategies (e.g., division of hundreds, tens, and units in mental mathematics). Monitoring is related to questions such as "Am I following my plan?" "Is this plan working?" "Should I use paper and pencil to solve the division?" and so on. In evaluation there is self-judging of the answer and of the process of getting to this answer.

Lucangeli and Cornoldi (1997) and Lucangeli, Cornoldi, and Tellarini (1998) disputed metacognitive beliefs as a separate component of metacognition and classified them within metacognitive knowledge (as support or hindrance and misconceptions or as a truly individual mathematical epistemology). Others have partly supported this view and defined these beliefs as non-(meta)cognitive but affective and conative (motivational or volitional) variables (e.g., Garcia & Pintrich, 1994; Masui & De Corte, 1999; McLeod, 1992; Vermunt, 1996). Simons (1996), however, described metacognitive beliefs as the broader general ideas and theories people have about their own and other people's cognition (e.g., on attribution, motivation, self-esteem) and regarded it as a third component of metacognition.

The debate on whether there are two (knowledge and skills) or three (knowledge, skills, and beliefs) components within metacognition remains unresolved (Dickson, Collins, Simons, & Kameenui, 1998). This debate is often based on theoretical concepts that lack empirical validation. Even authors who are in favor of a two-component approach to metacognition (e.g., Lucangeli & Cornoldi, 1997) have found it important to study attribution, not least because Pintrich and Anderman (1994) found that children with learning disabilities attribute success and failure to external factors and Borkowski, Teresa Estrada, Milstead, and Hale (1989) pointed out that all training programs on metacognition had to be combined with attributional retraining.

From a developmental point of view, metacognitive knowledge precedes metacognitive skills (Flavell, 1979; Flavell, Green, & Flavell, 1995; Flavell, Miller, & Miller, 1993). With age, children become increasingly conscious of cognitive capacities, strategies for processing information, and task variables that influence performance (Berk, 1997). Furthermore, low-effort skills (e.g., problem identification) precede high-effort skills (e.g., plan making and self-regulation; Berk, 1997; Shute, 1996). For a general review of the concept, we refer to Boekaerts (1999); Brown (1987); Hacker, Dunlosky, and Graesser (1998); Montague (1998); Simmons (1996); and Wong (1996).

In the last decade, various authors have described metacognition as essential in mathematics (Borkowski, 1992; Carr & Biddlecomb, 1998; De Clercq, Desoete, & Roevers, 2000; De Corte, Verschaffel, & Greer, 1996; De Corte, Verschaffel, & Op 't Eynde, 2000; Schoenfeld, 1992), although some authors have remained skeptical (e.g., Siegler, 1989). Metacognition was found to be instrumental in challenging tasks in mathematics, in not overtaxing the capacity and skills of children, and in relatively new strategies that are being acquired (Carr, Alexander, & Folds-Bennet, 1994; Carr & Jessup, 1995). Furthermore, especially during the initial stage of mathematical problem solving, when students build an appropriate representation of the problem, and in the final stage of interpretation and checking the outcome of the calculations, metacognition is involved in mathematical problem solving (Verschaffel, 1999). Metacognition prevents "blind calculation" or a superficial "number crunching" approach (e.g., answering "53" to the problem "50 is 3 more than ___" because more is always translated into addition) in mathematics (Vermeer, 1997, p. 23; Verschaffel, 1999, p. 218). Furthermore, metacognition allows students to use the acquired knowledge in a flexible, strategic way (Lucangeli et al., 1998).

**Aim and Research Questions**

Because metacognitive components include a wide range of overlapping phenomena (Boekaerts, 1999; Reder & Schunn, 1996), we have narrowed our research to three research questions. The present study aims to contribute some data to the debate on whether there are two or three components within metacognition. In order to do so, we investigate empirically in two exploratory studies whether some of the most used metacognitive parameters (declarative knowledge, conditional knowledge, procedural knowledge, prediction, planning, monitoring, evaluation, and attribution) can be combined into two (knowledge and skills) or three (knowledge, skills, and beliefs) supervariables on which young children differ. Of the metacognitive beliefs, we only include attribution because it seems important in children with learning disabilities and because it is often included in metacognitive training programs.

Because research on the relationship between metacognition and mathematics is usually conducted in older students (e.g., Montague, 1997) or in students with acquired deficits associated with brain injury (e.g., Mora & Saldana, 1995) and because inconsistent results were found in younger children (e.g., Siegler, 1989), we investigate whether the relationship between metacognition and mathematical problem solving can be found in elementary school children.

Furthermore, academic problems can be studied with either of two assumptions related to sample characteristics. A first key assumption is that there is a virtual continuum from very poor to very good mathematical problem solving. The first study was set up within this assumption to investigate our research questions within the empirical findings of our data set. In
Study 1, we investigate in a typical population whether children with below-average performance in the area of mathematics also show below-average performance on metacognition and whether age-matched children with high mathematics expertise exhibit general strengths on metacognition.

However, another key assumption is possible. Children with mathematics learning disabilities may also be considered as a clinical group of children with mathematical problem-solving scores below critical cutoff scores (1 SD, or below the 17th percentile). Study 2 was set up within this theoretical construct. To investigate whether the relationship between mathematics and metacognition also exists in children with an operational cutoff definition of mathematics learning disabilities, we have studied whether low metacognitive knowledge and skills and external attribution are core characteristics of young children with mathematics learning disabilities. We hypothesize that young children with specific mathematics learning disabilities will have less developed metacognitive knowledge, skills, and beliefs.

**STUDY 1**

**Method**

**Participants**

The participants, all third-grade students (ages 8–9), were referred to us by participating general education elementary schools. Each referred child was screened for inclusion in the study, with written parental consent, based on the following criteria:

1. no treatment for any kind of school-related problem;
2. average general intelligence level according to the school psychologist (Full Scale IQ between 90 and 120 on collective intelligence measurements); and
3. an overall school result of at least Level B (out of five levels: A–E).

Only white, native Dutch-speaking children without any history of severe reading problems, extreme hyperactivity, sensory impairment, brain damage, chronic medical condition, insufficient instruction, or serious emotional or behavioral disturbance were included as participants. The final sample included 80 third graders (31 boys and 49 girls).

The average score for the total sample on mathematical problem solving was percentile 56.82 ($SD = 33.07$). The average score on reading fluency was percentile 63.44 ($SD = 22.14$). No child with a reading score below the 25th percentile was accepted. Thus, children with severe reading problems were excluded, because some of the measures depended on the reading of instructions. The exclusion of children with reading disabilities narrows the scope of this study, but it also guarantees that any poor metacognitive results found are not due to problems in reading cognition.

As all the children were attending general education elementary school without reading or mathematics learning disabilities, according to teachers and parents, further individual intelligence assessments were not included. The socioeconomic status, based on the years of education of father ($M = 10.62$ years, $SD = 2.69$) and mother ($M = 10.62$ years, $SD = 2.90$), was recorded.

**Measures**

The Kortrijk Arithmetic Test (Kortrijke Rekenovest, KRT; Cracco et al., 1995) is a 60-item Belgian mathematics test on domain-specific knowledge and skills, resulting in percentile scores on mental computation and number system knowledge and in a total percentile score. The psychometric value has been demonstrated on a sample of 3,246 Dutch-speaking children. Because we found performances on mental computation (e.g., 129 + 879 = ___, and number system knowledge (e.g., add 3 tens to 61 and you have ___) on the KRT to be strongly interrelated in our sample, Pearson's $r = .76$, $p < .01$, we used the standardized total percentile score based on national norms.

The One Minute Test (Eén Minuut Test, EMT; Brus & Voeten, 1999) is a test of reading fluency for Dutch-speaking people, validated for Flanders on 10,059 children (Chesquière & Ruijsenaars, 1994). It measures the ability of children to read correctly as many words as possible out of 116 words (e.g., leg, car) in 1 minute.

The metacognitive tests were specifically designed for the present study and consisted of the Metacognitive Attribution Assessment (MAA) and the Metacognitive Skills and Knowledge Assessment (MSA). These instruments were tested in a pilot study ($n = 30$) in order to determine their usefulness for this age group and their sensitivity in measuring individual differences. Analyses showed that students without reading problems could handle the instruments well. Students were interviewed after the test about

1. the reasons they gave for certain predictions and evaluations;
2. their planning and monitoring following the prediction; and
3. the reasons they thought exercises to be difficult or easy.

The given answers all referred to the constructs in question. Moreover, different experts on mathematics and on metacognition were consulted in order to increase the construct validity. As to the reliability, Cronbach's $\alpha$ varied from .59 to .87. Furthermore, test–retest correlations of .81 ($p < .0005$) and interrater reliabilities for the metacognitive parameters varying between .98 and 1 ($p < .0005$) were found.

The MAA is a 13-item attribution rating scale based on the work of Carr and Jessup (1995; see Appendix A). Children evaluate internal stable (e.g., ability), internal nonstable (e.g., effort), external stable (e.g., task characteristics), and external nonstable (e.g., luck) attributions as causes of hypothetical situations. The four alternatives (internal...
stable, internal nonstable, external stable, and external nonstable) are ranked on a 4-point scale according to perceived importance (see Appendix A). The scores on internal nonstable (or effort) attribution were put into a composite score for this study. A Cronbach α of .59 was found.

The MSA was inspired by the work of Cross and Paris (1988), Myers and Paris (1978), Lucangeli and Cornoldi (1997), Lucangeli et al. (1998), and Montague (1997). The MSA assesses, without time limit, two metacognitive components (knowledge and skills) including seven metacognitive parameters (declarative, procedural, and conditional knowledge, and prediction, planning, monitoring, and evaluation skills; see Appendix B).

In the measurement of metacognitive declarative knowledge (15 items), children are asked to choose the easiest and the most difficult exercise out of five and to retrieve their own difficult or easy addition, subtraction, multiplication, division, or word problem. The exercises on procedural metacognitive knowledge (15 items) require children to explain how they solved exercises. Conditional metacognitive knowledge (10 items) is assessed by asking for an explanation of why an exercise is easy or difficult and asking for an exercise to be made more difficult or easier by changing it as little as possible. Children received 2 points for a correct and complete answer, 1 point for an incomplete but correct answer, and no points for any other answer.

In the assessment of prediction (25 items), children are asked to look at exercises without solving them and to predict whether they would be successful in this task on a 4-point rating scale (see Appendix B). Children might predict well and solve the exercise wrongly, or vice versa. Predictions corresponding with actual mathematics performance (rating “I am absolutely sure I can solve the exercise correctly” and correct answer, or rating “I am absolutely sure I cannot solve the exercise correctly” and incorrect answer) received 2 points. The rating “I am sure I can(not) solve the exercise correctly” and a corresponding mathematics performance received 1 point. Children were then scored on evaluation by doing the exercises on the same rating scale (see Appendix B). The answers were scored and coded according to the procedures used in the assessment of prediction skills. For planning, children had to put 10 sequences necessary to calculate (e.g., choose the appropriate strategy, read the assignment well, extract the information necessary for the solution) in order. When the answers were put in the right order, the children received 1 point. The following types of questions measured monitoring: “What kind of errors can you make doing such an exercise?” “How can you help younger children to perform well on this kind of exercise?” Complete and adequate strategies were awarded 2 points. Hardly adequate but not incorrect strategies (such as “I pay attention”) received 1 point. Answers that were neither plausible nor useful did not receive any points.

To examine the psychometric characteristics of the developed metacognitive parameters, Cronbach α reliability analyses were conducted. For declarative knowledge, procedural knowledge, and conditional knowledge, Cronbach αs were .66, .74, and .70, respectively. For prediction, planning, monitoring, and evaluation, Cronbach αs were .64, .71, .87, and .60, respectively.

Data Collection

All participants were assessed individually outside the classroom setting. They completed a standardized test on mathematics (the KRT), a reading fluency test (the EMT), and two metacognitive tests (the MAA and the MSA) on 2 different days, for a total of about 3 hours. The examiners, all trained psychologists, received 6 hours of theoretical and practical training in the assessment and interpretation of mathematics, reading, and metacognition.

Results

The sample was divided into three mathematics performance groups (below-average, average, and above-average performers) based on the standardized total percentile on the KRT. The criteria were based on the mean of at least 1 SD below the KRT mean were assigned to the group of below-average mathematical problem solvers. Thirty-nine children were assigned to the group of average mathematical problem solvers because their mathematics scores were between −1 SD and +1 SD. Twenty-six children obtaining a score equal to or exceeding 1 SD above the mean were assigned to the group of above-average mathematical problem solvers. Preliminary comparisons revealed that the three groups did not differ significantly in the socioeconomic level of the father, F(2, 77) = 0.06, p = .94, or the mother, F(2, 77) = 0.15, p = .86.

The mean total percentile scores on the KRT for the below-average, average, and above-average mathematical problem solvers were 8.73 (SD = 2.63), 52.82 (SD = 20.33), and 93.38 (SD = 6.30), respectively. The mean mathematical school grade of the below-average performers was 11.19% (SD = 5.73). The mean grades of average performers and above-average performers were 52.38% (SD = 19.49) and 91.12% (SD = 6.79), respectively. The means and standard deviations of the metacognitive parameters, all normally distributed, are presented in Table 1. The correlation matrix of these parameters is presented in Table 2.

Given the high intercorrelations between the metacognitive parameters, the internal structure of the data was analyzed with a principal components analysis to account for all the variance. This analysis was carried out to develop a small set of components empirically summarizing the correlations among the variables (see Note). To determine whether metacognitive parameters could be combined into two or three factor components, an initial run with principal components ex-
traction was carried out. Eight components were needed to account for all the variance in our data set. This initial number of eight could be reduced to three, retaining enough components for an adequate fit but not so many that parsimony was lost. This number of components in our solution was based on three criteria (Tabachnick & Fidell, 1996). The first criterion was that there were three components with eigenvalues higher than 1 (Kaiser normalization). Components 4, 5, 6, 7, and 8 had eigenvalues of 0.76, 0.63, 0.58, 0.36, and 0.32, respectively, and were found not as important from a variance perspective. The second criterion as to the adequacy of a two- of three-component solution to our data set was that a two-component solution accounted for 53.43% of the common variance, whereas a three-component solution explained 66.86% of the common variance. The third component accounted for 13.43% of the variance. The third criterion as to the number of components was the Cattell scree test of eigenvalues plotted against components. Again, there appeared to be three components in our data.

The component matrix is presented in Table 3. The eigenvalues (proportion of common variance) corresponding to Components 1 through 3 were 2.98 (37.33% of common variance), 1.24 (15.5% of common variance), and 1.09 (13.6% of common variance).

All weighted scores of the metacognitive parameters with loadings higher than .30 were added in the subsequent metacognitive components. Component 1 dealt with all metacognitive knowledge and skills parameters. Component 2 essentially dealt with off-line metacognitive activities either in the initial stage (prediction) or in the final stage (evaluation) of the mathematics performance. Component 3 dealt essentially with metacognitive beliefs about attribution, combined with some prediction. The residual correlations between Components 1 and 2, Components 1 and 3, and Components 2 and 3 were $r = .25$, $p < .05$; $r = .05$, ns; and $r = .28$, $p < .05$, respectively. We subsequently refer to these components as global metacognition, off-line metacognition, and attribution (see general discussion).

Given these components, we looked for between-group differences, expecting students performing below average on mathematics to have less global and less off-line metacognition and to attribute less to unstable and internal factors than their peers with above-average mathematical problem-solving skills.

To look for differences among students performing below average, average, or above average on mathematics, a multivariate analysis of variance (MANOVA) was conducted with global metacognition, off-line metacognition, and attribution as dependent variables and mathematical ability group membership as the independent variable. Post hoc analyses were conducted using the Tukey procedure, which corrects for unequal sample size. With an effect size of .50, we found a power of .80.

The MANOVA revealed a significant main effect for mathematical ability group on the multivariate level, $F(6, 150) = 7.78$, $p < .0005$. In the total model, metacognition was predicted for 42% ($1 - \text{Wilks's lambda}$) by the three mathematical ability groups, subsequently referred to as the degree of mathematical performance. Univariate significant between-group effects were found for global metacognition, off-line metacognition, and attribution (see Table 4). Global metacognition, off-line metacognition, and attribution were predicted for 16%, for 38%, and for 29%, respectively.

Post hoc follow-up analyses (see indexes in Table 4) revealed that above-average performers did better than average and below-average performers on global metacognition. No differences were found between below-average and average mathematical problem solvers on the global metacognitive component. All three performance groups also differed on off-line metacognition. Above-average mathematical problem solvers did better than average and below-average prob-
lem solvers, and average problem solvers did better than below-average mathematical problem solvers on off-line metacognition. Furthermore, above-average mathematical problem solvers had more internal attributions than average and below-average mathematical problem solvers. Means and standard deviations for the three mathematical ability groups on metacognition are presented in Table 4.

Discussion

Our results favored three metacognitive components (global metacognition, off-line metacognition, and attribution) that are different from the three forms of metacognition Simons (1996) described. Because these results did not validate a previously used metacognitive construct, it seemed useful to replicate these components in a sample of children with mathematics learning disabilities (see Study 2).

The findings from this study support the use of this assessment procedure on metacognition to differentiate among different groups of mathematical problem solvers in a continuum from very poor to very good mathematical problem solvers. We were able to differentiate among all three mathematics ability groups on off-line metacognition, confirming the importance of metacognition in the initial or forethought phase and in the final or self-reflection phase of mathematical problem solving (Verschaffel, 1999). Furthermore, above-average mathematical problem solvers had more global metacognition and higher internal and unstable attributions than average and below-average mathematical problem solvers without additional reading problems. Global metacognition and attribution did not, however, differ significantly between average and below-average mathematical problem solvers.

STUDY 2

In Study 2, we wanted to replicate the structure of the metacognitive components found in the random sample of Study 1 with children with specific mathematics learning disabilities from a cutoff perspective. Again, the exclusion of children with reading disabilities, and therefore the possible exclusion of children with both mathematics and reading learning disabilities, limits the findings, but it also guarantees that weaker metacognition scores in children with mathematics learning disabilities are not due to problems with reading the assignment.

In Study 1, a global score on mathematics (number system knowledge and mental computation) differentiated among children with above-average, average, and below-average mathematical problem-solving skills. Because Study 2 investigated metacognition in children with specific mathematics learning disabilities, we included a mathematics test on verbal numeral processing, as suggested by Lucangeli and Cornoldi (1997). We also included a test on retrieval of arithmetic number facts from semantic memory, because Geary (1993) discovered difficulties in this area in one subtype of children with mathematics learning disabilities. Furthermore, as the sample was no longer a random sample, IQ scores were added in the selection procedure to exclude the possi-
bility that some of the difference among the groups on the metacogni-
tive tasks would simply be due to dif-
fferences in level of intelligence.

Method

Participants

Fifty-nine children of average intelli-
gence with specific mathematics learn-
ing disabilities (22 boys and 37 girls) and 26 children (8 boys and 18 girls) who did not score above average in mathematics but did not have learning disabilities participated. The average age of the participants was 8.2 years (SD = 0.4). The sample was drawn, with the written consent of the children’s parents and teachers, from Grade 3 in several elementary schools. Participants were native Dutch-speaking children attending a general education elementary school and were selected for this study on the basis of teachers’ referrals and test scores indicating specific mathematics learning disabilities (LD) or not.

Teacher judgments were used be-
cause, although some researchers ques-
tion the trustworthiness of such data, reviews have indicated that these judg-
ments can serve as worthy assessments of students’ achievement-related be-
haviors triangulated with data gathered by other protocols (Winne & Perry, 2000). Furthermore, teachers’ perceptions of students’ use of strategies were found to be an important predictor of academic performance in children with LD (Meltzer, Roditi, Houser, & Perlman, 1998).

To be accepted in the cohort, the chil-
dren’s general intelligence had to be average according to the school psy-
chologist (Full Scale IQ between 90 and 120 on the WISC-R; Vander Steene et al., 1986) and the general school grade had to be at least a B level. Further-
more, the children’s reading perfor-
mances had to be rated 4 or 5 on a 7-
point performance rating scale (1 = very poor, 7 = very good) by the teacher. The mathematical problem-solving skills of the participants had to be rated 1 (children with severe math LD), 2 (children with moderate math LD), or 4 (moderate math performers) on the same scale. We did not include children with rates of 3 in order to differ-
entiate better between children with moderate math LD and moderate math performers without LD.

The average mathematics school grade for the total sample was 26.89% (SD = 16.20). The average score for the total sample on the KRT was percentile 18.14 (SD = 22.02). The average percentile scores on two other mathematica-


tical performance tests—the Arithmetic Number Fact Test (Tempo Test Reke-
en, TTR; de Vos, 1992) and the Word Problems Test (Vraagstukken, VT; Duda-
l, 1985)—were 30.13 (SD = 24.00) and 40.40 (SD = 25.03), respectively. The mean socioeconomic status of the father and mother (based on years of education) was 10.82 years (SD = 2.91) and 10.40 years (SD = 2.76), respectively.

Measures

The KRT was used to measure math abilities, as described in Study I. The MAA and MSA were adapted concern-
ing the number of items. Furthermore, two other mathematics tests (VT and TTR) and a teacher rating form (MSA ques-
nionnaire) were introduced.

The VT is a Belgian test to probe nu-
meral processing in 10 word problem formats (e.g., “John and Lisa together weigh 37 kg. John weighs 19 kg. What is the weight of Lisa?”). The psycho-
metric value has been demonstrated on a sample of 859 Dutch-speaking children.

The TTR is a test on 200 arithmetic number fact problems (e.g., 5 × 9 = . . . ). Children have to solve as many num-
ber fact problems as possible out of 200 in 5 minutes. The test has been normed for Flanders on 10,059 children (Chesnérié & Ruijsenaars, 1994).

The MSA questionnaire, which was created for this study, is a Likert-type rating scale 8-item questionnaire for teachers on metacognitive skills (e.g., the child never [1]/always [5] knows in


dvance whether an exercise will be easy or difficult). Furthermore, teachers rated the mathematical and reading perfor-
ences as well as the intelli-
gence of children (e.g., 1 = very low com-
pared to peers/7 = very good compared to peers).

The MSA questionnaire was tested in a pilot study in order to determine its usefulness for the purpose (Desoete & Roeyers, 2000; Desoete, Roeyers, & Buyse, 2000). Teachers were found to have a good picture of children’s perfor-
ances in the area of mathematical problem solving. All children with mathematics learning disabilities diagnosed by reliable and valid method-
ical problem-solving tests were also detected based on their teachers’ ratings (n = 150).

Because the number of items used in the MSA in Study 2 was adapted, the psychometric characteristics were ex-
amined again. All variables were nor-
mally distributed. Cronbach α reliabil-
ity analyses were conducted on the different metacognitive parameters. A Cronbach α of .70 was found for the MAA (10 items). A Cronbach α of .79 was found for declarative knowledge (25 items). A Cronbach α of .59 was found for procedural knowledge (20 items). A Cronbach α of .74 was found for conditional knowledge (40 items). For prediction (40 items), planning (20 items), monitoring (25 items), and evaluation (40 items), Cronbach αs were .87, .65, .70, and .90, respectively. The Cronbach α of the MSA ques-
nionnaire was .87. To examine the con-
current validity of the MSA, or the correspondence between the assessed metacognitive skills and the opinion of the teacher on the metacognitive skills of the participant, Cronbach α interreliability analysis was conducted with the four metacognitive skill scores (MSA) and four MSA questionnaire scores as scale items. This resulted in a Cronbach α of .70.

Data Collection

All participants were assessed individu-
ally outside the classroom setting by
skilled mathematical therapists who had received a 24-hour theoretical and practical training in the assessment of mathematics and metacognition. The children completed three standardized tests on mathematics (the KRT, the VT, and the TTR), as well as the MAA and the MSA, on 2 different days, for a total of about 4 hours. Teachers filled out a questionnaire on metacognitive skills, reading, mathematics, and intelligence (MSA questionnaire).

Results

The sample was divided into three mathematics ability groups based on mathematics standardized percentile scores (KRT, TTR, VT) and teacher referrals. Participants scoring at least 1 SD below the mean (or below the 17th percentile in mathematical ability) on at least two mathematics tests and below the 30th percentile in ability on the third math test were assigned to the group of children with a severe math disability if they also received a rating of 1 or 2 on mathematics on a 7-point scale according to the teacher. Most of these children performed more than 2 SD below the mean (or below the 3rd percentile) on all mathematics tests.

When participants received a rating of 2 on mathematics from the teacher and performed at least 1 SD below the mean (or below the 17th percentile in math ability) on one mathematics test and below the 30th percentile in ability on the other math tests, they were assigned to the group of children with a moderate disability. Participants obtaining a score of -0.5 SD below or +0.5 SD above the mean on all three mathematics tests and a mathematics rating of 4 by the teacher were assigned to the group of average performing children without disabilities.

Preliminary comparisons revealed that the three groups did not differ significantly in the socio-economic status of the father, F(2, 82) = 1.55, p = .22, or the mother, F(2, 82) = 2.16, p = .12. To exclude the possibility that some of the difference between the groups on the metacognitive tasks was due to IQ differences, the mean IQ scores of the three groups were compared. As shown in Table 5, no differences on IQ or socioeconomic status were found among the three mathematical problem-solving performance groups. Furthermore, descriptive statistics with mean ratings on the mathematics tests (KRT, VT, and TTR) for the children with a severe math disability, a moderate math disability, and no math disability are also presented in Table 5.

A principal components analysis was carried out to explore the internal structure of the metacognitive data and to find out whether the metacognitive parameters (declarative knowledge, conditional knowledge, procedural knowledge, prediction, planning, monitoring, evaluation, and attribution) could be combined into the same supervariables as in Study 1. Eight components were needed to account for all the variance in our dataset. Again, this initial number of eight could be reduced to three components based on the Kaiser normalization, the additional variance of the third component, and the Cattell scree test. Components 4, 5, 6, 7, and 8 had eigenvalues of 0.82, 0.58, 0.56, 0.37, and 0.27, respectively, and were not important from a variance perspective. Furthermore, the third component had an additional explained variance of 12.92%, and the Cattell scree test confirmed this three-component solution.

Between Components 1 and 2, 1 and 3, and 2 and 3, correlations of r = .62, p < .0005; r = .03, ns; and r = .00, ns, respectively, were found. The means and standard deviations for the metacognitive components are presented in Table 6. As the metacognitive components are intercorrelated, the correla-
The intercorrelation matrix is presented in Table 7. The three-component solution (see Table 8) was comparable to the one found in Study 1 (global metacognition, off-line metacognition, and attribution) and explained 67.5% of the common variance.

The eigenvalues (proportion of common variance) corresponding to Components 1 through 3 (see Table 8) were 3.04 (38.03% of common variance), 1.32 (16.52% of common variance), and 1.03 (12.92% of common variance), respectively. All weighted components with their loadings, if higher than .30, were added in the subsequently used global, off-line, and attribution components.

We also looked for differences between children on metacognition. A MANOVA was conducted with global metacognition, off-line metacognition, and attribution as dependent variables. The variable differentiating among children with severe math LD, children with moderate math LD, and children without LD was used as the independent variable. Post hoc analyses where conducted using the Tukey procedure, which corrects for unequal sample size. With an effect size of .50, we found a power of .85. The MANOVA (see Table 9) revealed a significant main effect for mathematics learning disabilities, $F(6, 160) = 16.40, p < .0005$.

In the total model, off-line and global metacognition were predicted for 26% and for 55%, respectively. The model did not significantly predict the attribution score. Significant between-subject effects were found for the degree of mathematics learning disability on the global metacognition and off-line metacognition components, but no significant results were found on attribution. Descriptive statistics with mean ratings for children with severe math LD, moderate math LD, and without a math LD are presented in Table 9.

Post hoc follow-up analyses revealed that children with severe math disabilities performed worse than children with a moderate disability or average performers without disabilities on global and off-line metacognition (see indexes in Table 9). Participants with a moderate disability did not differ significantly from average mathematical problem solvers without LD on global metacognition, but they did significantly worse than average mathematical problem solvers without LD on off-line metacognition (see indexes in Table 9).

### Discussion

In this selected sample of children with specific mathematics learning disabilities, our results indicated three metacognitive components similar to those found in the first study as internal structure of the data. All metacognitive knowledge parameters were combined with all metacognitive skills in the first, global metacognitive component. The off-line skills (prediction and evaluation) were combined with a negative loading on monitoring in the second component (off-line metacognition). The

### Table 7

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>2. Conditional knowledge</td>
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<td>—</td>
<td>—</td>
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<td>3. Procedural knowledge</td>
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<td>—</td>
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<td>5. Planning</td>
<td>.33*</td>
<td>.23</td>
<td>.22</td>
<td>.12</td>
<td>—</td>
<td>—</td>
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<td>6. Monitoring</td>
<td>.53**</td>
<td>.39**</td>
<td>.46**</td>
<td>.13</td>
<td>.41**</td>
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<td>—</td>
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<td>.30*</td>
<td>.29*</td>
<td>.67**</td>
<td>.30*</td>
<td>.17</td>
<td>—</td>
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<td>8. Attribution</td>
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<td>.07</td>
<td>.08</td>
<td>.02</td>
<td>.20</td>
<td>.06</td>
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### Table 8

<table>
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<td>-.48</td>
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<td>.66</td>
<td>.02</td>
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<tr>
<td>Eigenvvalue</td>
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<td>1.32</td>
<td>1.03</td>
</tr>
<tr>
<td>% of variance</td>
<td>38.03</td>
<td>16.52</td>
<td>12.92</td>
</tr>
<tr>
<td>Total group</td>
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<td>24.88</td>
<td>21.74</td>
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<tr>
<td>M</td>
<td>22.51</td>
<td>11.77</td>
<td>4.92</td>
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</tbody>
</table>

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attrition on effort, combined with a negative loading on planning, created the third component.

Furthermore, participants with a severe specific mathematics disability (and intact reading skills) showed less global metacognition than their peers with a moderate or no specific mathematics learning disability. Off-line metacognition differed among all three groups. Participants with specific mathematics learning disabilities performed significantly lower than average mathematical problem solvers on off-line metacognition. Furthermore, children with a severe specific mathematics learning disability performed worse on off-line metacognition than their peers with a moderate specific mathematics learning disability. No between-group differences where found on attribution.

**GENERAL DISCUSSION**

Since the introduction of the concept of metacognition, there has been considerable debate about the multiple meanings of the concept (Boekaerts, 1999; Desoete, Roeyers, Buysse, & De Clercq, in press). Our two exploratory studies investigated whether (declarative, procedural, and conditional) metacognitive knowledge, metacognitive skills (prediction, planning, monitoring, and evaluation), and metacognitive attribution could be combined into a smaller number of supervariables, validating a three- (knowledge, skills, and conceptions) or two-component (knowledge and skills) construct. Moreover, we looked for differences in metacognition between students with and without mathematics learning disabilities in order to investigate whether metacognition should be part of the assessment of children with mathematics learning disabilities.

In both studies, we failed to validate the traditionally used components of metacognition (knowledge, skills, and beliefs) related to successful execution of mathematical problem solving. We did find three components, but not the expected ones. Instead, three different metacognitive components combined the metacognitive parameters into a smaller number of supervariables in both studies.

All metacognitive knowledge parameters (declarative, conditional, and procedural) were found to be interrelated with all metacognitive skills (prediction, planning, monitoring, and evaluation). Because this first component combined all metacognitive parameters with the exception of the contested belief component of metacognition (Lucangeli & Cornoldi, 1997), we labeled the component as global metacognition, including both on-line and off-line measured metacognitive aspects.

Prediction and evaluation were found to be interrelated (Component 2). As both these metacognitive parameters were measured either before or after the solving of exercises, we labeled this metacognitive component off-line (measured) metacognition, in contrast to on-line measured metacognitive skills. Monitoring was found to be negatively correlated with off-line metacognition.

Metacognitive attribution was detected as a different component (Component 3). In Study 1, attribution on effort was related to high off-line prediction skills, whereas in Study 2 attribution on effort was found to be correlated with low on-line planning behavior. Because the loading on attribution was very high in both studies and the combination with other parameters (low procedural knowledge and high prediction skills in Study 1, and low planning skills in Study 2) was not stable, we labeled this component as attribution. In both studies, we found significant correlations between global and off-line metacognitive components.

These results indicate the existence of a construct for prediction and evaluation skills (Component 2) that, although related, is somehow different from the construct combining these skills with planning and monitoring skills and metacognitive knowledge (Component 1). These findings are consistent with the research of Verschaffel (1999), who stressed the importance of metacognition during the initial stage (prediction) of mathematical problem solving, before the actual (on-line) calculation. Furthermore, metacognition was also found to be important in the final stage (evaluation) of mathematical problem solving, after the actual on-line calculation. Therefore, these metacognitive activities take place without children actually calculating and can be considered as off-line metacognitive in nature.

Our research also offered some insights into the relationship between metacognition and mathematics in
young elementary school children. Both studies have shown metacognition to be characteristic for the above-average (expert) approach to mathematical problem solving in the elementary school. In Study 1, the importance of metacognition in mathematical problem solving could be demonstrated in a random sample of third-grade students. Above-average mathematical problem solvers (experts) had more global and off-line metacognition and attributed failure and success more to internal and unstable (effort) causes than average and below-average mathematical problem solvers (novices). In Study 2, the relevance of metacognition could be confirmed in third-grade students with specific mathematics learning disabilities from a cutoff perspective. Average mathematical problem solvers without learning disabilities did better on global and off-line metacognition than their age- and intelligence-matched peers with a severe specific mathematics learning disability. Furthermore, children with a severe mathematics learning disability had lower off-line metacognition scores than their peers with a moderate mathematics learning disability.

To assess whether impairments in the three metacognitive components (global, off-line, and attribution) were core characteristics of specific mathematics learning disabilities, both studies were analyzed on the difference between children with and without mathematics learning disabilities. No conclusive evidence was found for a global metacognitive deficit (Component 1) because children with different mathematical problem-solving skills did not always differ significantly on global metacognition. In Study 1, we could not differentiate between average and below-average mathematical problem solvers on global metacognition, whereas in Study 2 no significant differences in global metacognition were found between participants with a moderate mathematics learning disability and their average performing peers without mathematics learning disabilities.

Off-line metacognition (Component 2), however, seemed especially important because the three performance groups in both studies differed on this component. In Study 1, children with below-average mathematical problem-solving skills had lower off-line metacognitive scores than their peers with average mathematical problem-solving skills. Moreover, children with average mathematical problem-solving skills did worse than their peers with above-average mathematical problem-solving skills. In Study 2, children with severe mathematics learning disabilities had less developed off-line metacognitive skills than their peers with moderate mathematics learning disabilities. Both groups did worse than children with average mathematical problem-solving skills without mathematics learning disabilities.

A less developed attribution on effort (Component 3) was found not to be a core characteristic of children with mathematics learning disabilities in our sample, as we failed to find differences between subgroups of children with and without specific mathematics learning disabilities in Study 2. Above-average performers, however, attributed significantly more to effort than average and below-average performers faced with mathematical problem-solving tasks in Study 1.

These results should be interpreted with care because metacognitive skills might involve different mental operations (e.g., simultaneous versus serial thinking) and might be age dependent and still maturing until adolescence (Berk, 1997). Furthermore, because the MAA and MSA depended on children reading the instructions, only children of average intelligence without additional reading problems were included in these studies. Thus, there is a possible exclusion of children with combined mathematics and reading learning disabilities, a subtype described by Geary (1993) as children having difficulties in fact retrieval. The empirically demonstrated metacognitive components, therefore, still need a full explanation from more applied research on different age, reading, and intelligence groups. To exclude alternative possible explanations, our studies need to be replicated with a larger sample of children with mathematics learning disabilities. It would also be useful to compare off-line metacognition in children with specific mathematics learning disabilities and intact reading skills with metacognitive performances of children with specific reading disabilities and intact mathematical problem-solving skills and to investigate the modifiability of metacognitive performances. Such studies are currently being prepared.

In summary, our studies suggest that three metacognitive supervariables are involved in mathematical problem solving in Grade 3. These components can help to gain a better understanding of contributors to successful mathematical performance. Furthermore, the findings from these studies support the use and importance of a metacognitive assessment procedure to differentiate among mathematical ability groups and between students with and without specific mathematics learning disabilities. However, despite the consistency of the findings in these studies, only off-line metacognition (prediction and evaluation) could differentiate between average and below-average mathematical problem solvers and between children with a severe and children with a moderate mathematics learning disability. Taking into account the complex nature of mathematical problem solving, it may be useful to assess off-line metacognition in young children with mathematics learning disabilities in order to focus on these factors and their role in mathematics learning and development.

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AUTHORS’ NOTES

1. We thank Dr. Daniella Lucangeli for her important conceptual contribution to this research.

2. This study was supported by the Stichting Integrierte Gehandicapten (SIG), the Arteselde College Ghent, and the Centrum ter Befordering van de Cognitieve Ontwikkeling (CoBCO), to whom the authors extend their thanks.

NOTE

With a principal axis factor analysis, allowing co-variance within the data, the same three factors were found and the data remained almost the same.

REFERENCES


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APPENDIX A
Sample Item From the Metacognitive Attribution Scale

Read the following statements and rank them (in □) as
4  the most important reason
3
2
1  not an important reason at all

Chris cannot solve word problems.

This is because . . . (attribution of failure)

☐ The teacher did not explain the word problems enough this time. (external nonstable)
☐ Word problems are always difficult. (external stable)
☐ Chris did not try hard enough. (internal nonstable)
☐ Chris is not good at mathematics. (internal stable)

APPENDIX B
Sample Items From the Metacognitive Skill and Knowledge Assessment (MSA)

Look at these additions (without solving them).

45 + 28 =
45 + 23 =
43 + 8 =
23 + 6 =
9 + 23 =

• Which addition is the most difficult one?  (declarative metacognitive knowledge)
• Why?  (conditional metacognitive knowledge)
• How will you proceed?  (procedural metacognitive knowledge)

Look at this exercise (without solving the exercise).
25 is 1 more than ?

Can you solve this exercise correctly?  (metacognitive prediction skill)

☐ I am absolutely sure I can solve the exercise correctly.
☐ I am sure I can solve the exercise correctly.
☐ I am sure I cannot solve the exercise correctly.
☐ I am absolutely sure I cannot solve the exercise correctly.

How will you proceed to solve this exercise? Put the sentences in the correct order.
25 is 1 more than ?  (metacognitive planning skill)

☐ Choose the appropriate strategy.
☐ I read the assignment well.
☐ I extract the information necessary for the solution.

Do it. Solve the exercise.
25 is 1 more than ?
You have answered. Are you sure that your answer is the correct answer? (metacognitive evaluation skill)

☐ I am absolutely sure I have solved the exercise correctly.
☐ I am sure I have solved the exercise correctly.
☐ I am sure I have not solved the exercise correctly.
☐ I am absolutely sure I have not solved the exercise correctly.

• According to you, what kind of mistakes do children make in such exercises? (metacognitive monitoring skill)
• What is important, according to you, to succeed in subtraction? (metacognitive monitoring skill)

☐ to put the numbers in the right place
☐ to know the multiplication tables well
☐ to pay attention to tens and units
☐ to finish as soon as possible

Write in ☐: 4 the most important reason

3
2
1 not important at all

• How can you help young children with these kinds of exercises? (metacognitive monitoring skill)

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