Secondary students with learning disabilities generally make inadequate progress in mathematics. Their achievement is often limited by a variety of factors, including prior low achievement, low expectations for success, and inadequate instruction. This article will discuss techniques that have been demonstrated to be effective with secondary students who have learning disabilities in mathematics.

The body of research on mathematics instruction for secondary students with learning disabilities (LD) is not developed well enough to describe a specific and comprehensive set of well researched practices, but it is sufficient for defining a set of procedures and issues as clearly associated with effective instruction and increased student achievement. In this article, data-based investigations of procedures that have evaluated the effectiveness of mathematics instruction with secondary students with LD will be discussed. Although this discussion will be based primarily on studies that were limited to secondary students with LD, some research on the instruction and achievement of younger students or higher achieving populations will be considered.

This discussion considers six factors that predictably confound efforts to increase the effectiveness of instruction. Each of the factors is particularly relevant in the case of instruction for secondary students with LD. These factors are (a) students' prior achievement, (b) students' perceptions of self-efficacy, (c) the content of instruction, (d) management of instruction, (e) educators' efforts to evaluate and improve instruction, and (f) educators' beliefs about the nature of effective instruction.

**Prior Achievement**

Although students with LD spend a substantial portion of their academic time working on mathematics (Carpenter, 1985), severe deficits in mathematics achievement are apparent and persistent. Although secondary students with LD continue to make progress in learning more complex mathematical concepts and skills, it appears that their progress is very gradual (Cawley, Fitzmaurice, Shaw, Kahn, & Bates, 1979; Cawley & Miller, 1989). McLeod and Armstrong (1982) surveyed junior high, middle school, and high school math teachers regarding mathematics achievement. The teachers reported that skill deficits in basic computation and numeration were common. Specifically, McLeod and Armstrong found that secondary students with LD had difficulty with basic operations, percentages, decimals, measurement, and the language of mathematics. Algozzine, O'Shea, Crews, and Stoddard (1987) examined the results of 10th graders who took Florida's minimum competency test of mathematics skills. Compared to their general class peers, the adolescents with LD demonstrated substantially lower levels of mastery across all subtests. Algozzine et al. reported that the students with LD consistently scored higher on items requiring the literal use of arithmetic skills than on items requiring applications of concepts. Similarly, the results of the National Assessment of Educational Progress (cited in Carpenter, Matthews, Linquist, & Silver, 1984) clearly indicated that too many students in the elementary grades failed to acquire sufficient skills in operations and applications of mathematics. These persistent skill deficits, combined with limited fluency of basic fact recall (i.e., lack of automaticity), will hinder the development of higher level mathematics skills and will compromise later achievement (Hasselbring, Coln, & Bransford, 1988).

For secondary students with disabilities, the adequacy of instruction in mathematics will be judged not merely on how quickly basic skills can be learned. Students must also acquire generalizable skills in the application of mathematical concepts and problem solving. The task of designing instructional programs that result in adequate levels of acquisition and generalization for students who have experienced seriously low levels of achievement for a large proportion of their academic careers is indeed a challenging one.

**Perceptions of Self-Efficacy Attributes for Failure or Success**

Individual differences in cognitive development certainly affect the achievement of academic skills. In earlier years, many professionals readily accepted that individual psychological differences accounted for failure to learn in school. Currently, a more parsimonious explanation is that many students fail as a result of ineffective instruction (Engelmann & Carnine, 1982; Kameenui &
Students’ expectations for failure frequently develop as a result of prolonged experiences with instruction that fails to result in successful performance.

By the time students with LD become adolescents, they have typically endured many years of failure and frustration. They are fully aware of their failure to achieve functional skills in the operations and applications of mathematics. Although research supports the argument that perceptions of self-efficacy are task specific and fairly accurate, it does not reveal that learning disabilities are consistently associated with lower general self-concepts (Chapman, 1988). Chapman concluded that students who come to doubt their abilities (a) tend to blame their academic failures on those deficits, (b) generally consider their low abilities to be unchangeable, (c) generally expect to fail in the future, and (d) give up readily when confronted with difficult tasks. Unless interrupted by successful experiences, continued failure tends to confirm low expectations of achievement, which in turn sets the occasion for additional failure.

Pajares and Miller’s (1994) study of self-efficacy and mathematics has implications for teachers who attempt to remediate low math achievement. They found that students’ judgments of their ability to solve specific types of mathematics problems were useful predictors of their actual ability to solve those problems. Specific student estimates of self-efficacy were more accurate predictors of performance than prior experience in mathematics. Extending the results suggests three important issues. First, judgments of self-efficacy are task specific and generally accurate. Second, student judgment of self-efficacy may provide insights that will be valuable supplements to teacher assessments of performance skills. Third, negative expectations and motivational problems may be reduced by interventions to eliminate deficits in specific mathematics skills. The fact that student ratings of self-efficacy are accurate (for both successful and unsuccessful students) suggests that this is not a chicken-or-the-egg-type issue. Instead, it suggests that initially there is a fairly direct path from instruction to performance and, subsequently, to perceptions of self-efficacy. When instruction is effective (i.e., when students master targeted competencies), performance is enhanced and an accurate and positive perception of self-efficacy results. Conversely, ineffectivе instruction leads to poor performance and an accurate but negative estimate of self-efficacy.

For secondary students with LD, expectations to fail to learn mathematics skills can be an important obstacle. Although students’ perceptions of their self-efficacy originally develop from their experiences with success and failure in instruction, those expectations later can become factors that prevent low-achieving students from attempting to learn, or persevering in trying to apply, mathematical concepts and skills.

Content of Instruction

With high-quality instruction, students will acquire skills in less time and make more adaptive generalizations than they would with lower quality instruction. Quality of instruction is dependent on two elements of curriculum design: organization of content and presentation of content. Secondary students are expected to master skills in numeration and mathematical operations and to be able to apply those skills across a broad range of problemsolving contexts. Adolescents with LD are not likely to acquire adequate competence unless the content of their instruction is carefully selected and organized. Woodward (1991) identified three empirically supported principles related to curriculum content that contribute to the quality of instruction: the nature of examples, explicitness, and parsimony.

Nature of Examples

Students learn from examples. An important part of the business of education is selecting and organizing examples to use in instruction such that students will be able to solve problems they encounter outside of instruction. Unfortunately, commercial math curricula frequently do not adequately manage the selection or organization of instructional examples. Two deficiencies that contribute to inefficient instruction and chronic error patterns in the management of instructional examples are common to commercial math curricula. First, the number of instructional examples and the organization of practice activities are frequently insufficient for students to achieve mastery (Silbert, Carnine, & Stein, 1990). As a result, although high-achieving students may quickly attain near-perfect performance, low-achieving students (including students with a variety of learning difficulties) fail to master the same math skills before teachers move on to new instructional tasks. Empirical studies of two types support the validity of this phenomenon. First, classroom observations of teacher behavior indicate that teachers tend to direct their instruction (in terms of the difficulty of the material) to students of high-average achievement (Brophy & Good, 1974). Second, classroom observation indicates that when students with LD are taught in mainstream settings with students without disabilities, they average only 60% correct (Chow, 1981), a figure considerably below the performance levels of 90% to 100% correct that most educational experts require for task mastery.

A second deficiency is an inadequate sampling of the range of examples that define a given concept. If some instances of a concept are under represented in instruction or simply not included in instruction, students with LD will predictably fail to learn that concept adequately. The direct connection between the range of examples and task mastery has been demonstrated in instructional areas including teaching fractions (Kelly, K Gersten, & Carnine, 1990) and test taking (McLone, Scruggs, Mastropieri, [ & Zucker, 1986). The adequacy of a selection of examples depends on several factors, including (a) possible variations of the concept, (b) the likelihood that irrelevant or misleading variables will be erroneously associated with the concept, (c) the complexity of the concept being taught, and (d) the variety of potential applications of the concept.

If inadequate selections of instructional examples are provided, the range of a concept is not illustrated and students form limited or erroneous conceptualizations. Silbert et al. (1990) provided an example of instruction in the analysis of fractions that predict ably contributes to limited understanding of fractions: Generally, students are taught that fractions represent equal divisions of one whole, and during the elementary grades, considerable instruction focuses on teaching the concept of a fraction, with examples such as 1/2, 1/4, 1/8, and so forth. This representation of the concept of a fraction is inadequate, however, because not all fractions are less than one whole unit. Some fractions are equal to or greater than 1 (e.g., all improper fractions, such as, 6/5, where the numerator is larger than the denominator). Representing the concept of fraction as a quantity less than one whole limits student understanding of the wider range of possible fraction concepts. An inadequate conceptualization of fractions contributes to inadequate understanding of computation with fractions and, thus, severely limits problem-solving skills. For example, students who
Learn" that fractions represent quantities less than one unit may experience difficulty understanding that even the quantity represented by a proper fraction (i.e., one in which the numerator is smaller than the denominator) may not be less than 1. Consider an example involving the fact that 35 of the students in a class of 25 are female. In this case, the quantity represented by 5 of the class equals 15 individuals. To ensure that this example of the concept of a fraction is understood, teachers must include examples whereby fractions represent portions of groups as well as portions of single objects.

To learn correct conceptualizations, students must be taught which attributes are relevant and which are irrelevant. If sets of instructional examples consistently contain attributes that are irrelevant to a concept, then students will predictably learn misconceptualizations that may seriously hinder achievement. It is not uncommon to find that presentations of misleading variables have inhibited mathematics achievement. One important illustration is the frequent use of key words in story problems (e.g., Carpenter et al., 1984; Nesher, 1976; Wright, 1968). Although so-called key words frequently appear in story problems and often indicate the solution, they are not relevant per se to the solution of the problems. Unfortunately, many low-achieving students learn to depend on key words instead of attending to the more critical information presented in the problem. Consequently, they are apt to experience difficulty solving problems that do not have key words, or that use key words in ways that are irrelevant to any mathematical solution.

Such misconceptualizations will confuse students unless their exposure to potentially misleading cues is carefully managed. Initial sets of instructional examples should not contain key words. As students become proficient with those examples, problems that contain relevant key words can be included in instructional sets, juxtaposed with examples in which the key words are irrelevant to the solution (see Engelmann & Carnine, 1982; Kameenui & Simmons, 1990; Tennyson & Park, 1980).

Secondary students must learn to deal with complex notations, operations, and problem-solving strategies. Complexity is sometimes related to the level of abstraction the student must deal with; it may also be related to understanding the relationships between associated concepts. Thus, instructional examples should provide for the systematic progression from concrete to more abstract representations (Mercer & Miller, 1992) and from simpler to more involved relationships among concepts and rules. Students with LD have difficulty with complex tasks because they do not receive sufficient opportunities to work with complex instructional examples. Although instruction on simple forms of concepts or strategies does not facilitate generalization to more complex forms without additional instruction, Rivera and Smith (1988) demonstrated that carefully managed instruction for solving more complicated forms of division problems facilitated students’ independent solutions of simpler division problems.

Explicitness

Explicitness of curriculum design refers to the unambiguous presentation of important concepts and skills and the relationships among them (Woodward, 1991). The explicitness of a curriculum is affected by the quality of teacher decisions made and actions taken at the following five stages of the instructional design process:

- Determining the concepts and skills that must be learned;
- Identifying the important relationships among concepts and skills;
- Organizing facts, concepts, and skills into logical hierarchies;
- Developing sets of instructional examples that unambiguously illustrate the range of concepts and skills that must be mastered;
- Presenting the instructional examples to the student.

Ambiguities that enter the design process will result in predictable misunderstandings. Although research in this area is limited, Jitendra, Kameenu, and Carnine (1994) demonstrated that a highly explicit math curriculum produced greater student achievement than a less explicit curriculum.

The premise that curriculum quality is related to the degree to which concepts and skills are explicitly taught is being debated in the current movement to reform mathematics education. Instead of the argument for unambiguous organization and presentation, an increasingly popular position is that students should participate in instructional activities that allow them to construct both knowledge of concepts and skills and understandings of interrelated hierarchies (e.g., Poplin, 1988a, 1988b). Others contend that although students construct their own knowledge, teachers can contribute to the efficiency of instruction by carefully planning and structuring the learning experience (e.g., Harris & Graham, 1994; Mercer, Jordan, & Miller, 1994; Pressley, Harris, & Marks, 1992). Engelmann (1993) acknowledged that it is logically impossible to have universal hierarchies of instructional skills, but he also contended that curricula must be organized around explicit instructional priorities. According to Engelmann, if hierarchical sequences are not developed around explicit instructional priorities, it is unlikely that students with learning difficulties will progress efficiently.

We encourage practitioners to follow this debate in the professional literature, because there is likely to be a subsequent effect on classroom practice. However, for students with LD, we believe that this dictum summarizes the empirical literature: More explicit instruction results in more predictable, more generalizable, and more functional achievement. If we do not explicitly teach important knowledge and skills, these objectives will not be adequately learned.

The need for explicitly identifying and teaching important mathematical concepts, skills, and relationships is apparent in the persistent failure of students with LD to deal adequately with common fractions, decimal fractions, percentages, ratios, and proportions. A clear example of the link between explicit instruction and expected student performance can be seen in states that use proficiency tests as criteria for promotion or graduation. In the state of Ohio for example, students in the ninth grade take basic academic-proficiency examinations. They must pass these exams in order to graduate from high school. The Ohio Department of Education provides funds for school districts to deliver remedial programs to students who fail the proficiency examination. The remedial programs provide explicit instruction on the types of problems and relationships presented on the test. Although one might question the wisdom or utility of proficiency testing, the telling argument here is that when educators were forced to confront the
practical problem of designing the most efficient and effective means for students to achieve criterion performance (i.e., pass a test), an explicit approach was selected.

Parsimony

Effective curricula provide for an economical, or parsimonious, use of time and resources. Woodward (1991) contended that emphasis should be given to mastery of concepts, relationships, and skills that are essential for the subsequent acquisition and functional generalization of math skills. Curricula should be organized so that instruction of specific skills and concepts is tightly interwoven around critical concepts. Woodward's test for the parsimony of an instructional program is whether or not what is learned at one time will be used later. In general, practitioners are well aware of the need for functional instruction, because they must frequently answer questions such as, “Which is more important to the students, a unit on metric conversion or one on checking account management?” or “How often in life will students with LD be required to divide fractions?” However, it is not enough to identify information and skills that take the highest priority. The curriculum must be organized so that the greatest amounts of high-priority information and skills can be mastered as efficiently as possible.

In summary, the content of instruction and its organization play critical roles in determining its quality and outcomes. Unfortunately, commercial math curricula frequently stop well short of providing adequate opportunities to learn to solve mathematics problems that involve the contexts of work and everyday situations. Successful student performance on algorithms and abstracted word problems does not always result in competent real, life mathematical problem solving. In response to that problem, Hasselbring and colleagues (Botte & Hasselbring, 1993; Bransford, Sherwood, Hasselbring, Kinzer, & Williams, 1990; Cognition and Technology Group at Vanderbilt University, 1991; Hasselbring et al., 1991) have been conducting innovative research on teaching adolescents with LD to solve contextualized mathematical problems. They use video disks and direct instruction techniques to present complex mathematical problems that are embedded in portrayals of real-life situations. The results of their studies are encouraging and clearly demonstrate that adolescents with LD can be taught to solve more complex problems than their teachers generally expect from them. On the other hand, those studies also demonstrate that unless students are guided in solving complex math problems and are also given sufficient opportunities to independently attempt to solve such problems, they usually will not learn adequate generalized problem solving skills.

Management of Instruction

Archer and Isaacson (1989) listed three variables that can be measured to evaluate quality of instruction at any grade level: time on task, level of success, and content coverage. According to their perspective, good teachers manage instruction so that students (a) spend the major portion of instructional time actively engaged in learning, (b) work with high levels of success, and (c) proceed through the curriculum while acquiring increasingly more complex skills and important generalizations. Thus, good teaching is indicated by students' responses to instruction. Obtaining high levels of achievement requires effective management of instruction.

Zigmond (1990) reported data from observational studies indicating that too many teachers of secondary students with LD may not be managing instruction effectively. She reported that during their class periods, resource room teachers spent slightly less than 40% of class time in instructional interactions. Teachers spent 28% of their time “telling students what to do, but not teaching them how to do it, and another 23% of the time not interacting with students at all” (p. 6). Although students were observed to be on task for about three quarters of the class period, they were often completing worksheets. Worksheets may be appropriate for practice, but they are not useful for introducing new information and skills. The lecture format of instruction that often occurs in general secondary mathematics classes often is not effective, either. Zigmond observed that many large classes are not managed well-students who have difficulty understanding what the teacher is talking about tend to be off task, and misunderstandings often go undetected.

Alternatives to worksheet instruction and didactic lectures have been investigated in empirical studies. The approaches to managing instruction that we will discuss are direct instruction, interactive instruction, peer-mediated instruction, and strategy instruction. The effectiveness of the first three of these approaches has been documented across a variety of curriculum areas with secondary students in general, remedial, and special education programs.

Direct Instruction

There are a variety of interpretations of the term direct instruction. In some discussions it refers to a model for both curriculum organization and presentation procedures (e.g., Becker & Carnine, 1981; Engelmann & Carnine, 1982; Gersten, 1985). Other descriptions of direct instruction refer to a set of procedures for actively involving students in academic learning (Christenson, Ysseldyke, & Thurlow, 1989; Rosenshine, 1976; Rosenshine & Stevens, 1986). In both discussions, instruction is teacher-led and characterized by (a) explicit performance expectations, (b) systematic prompting, (c) structured practice, (d) monitoring of achievement, and (e) reinforcement and corrective feedback.

Archer and Isaacson (1989) provided a structure for teacherled instruction that is divided into three phases: the lesson opening, the body of instruction, and the closing of the lesson (see Table 1). During each phase the teacher works to maintain high levels of active student involvement, successful acquisition, and progress through the curriculum.

The Opening of the Lesson.
The teacher first gains the attention of the students. A brief statement, such as, "Look up here. We are going to begin," is generally adequate. The teacher reminds the students of what was accomplished in the previous lesson and sets the goal for the current session. For example, "Yesterday we learned how to calculate the area of squares and rectangles. Today we are going to learn how these calculations are used in the home to lay carpet, paint walls, and tile floors." Lengthy reviews or previews of upcoming lessons are unnecessary. If this part of the lesson is not brief, students are likely to start attending to other things.

TABLE 1: Phases of Structured Academic Presentations
are confronted with loosely or ranged instructional procedures where the chances that they will fail are high. Finally, there is instructional prompts is reduced. Thus, the students learn to expect to be successful and are less apt to resist instruction than if they chance that they will guess and make errors. Because prompts are faded, the chances that students will develop dependence on for efficient instruction across age levels, ability levels, and curriculum domains. If students imitate the initial prompts, there is little older students with LD (e.g., Perkins & Cullinan, 1985; Rivera & Smith, 1988).

Several empirical evidence in the professional literature that direct instruction procedures have been effectively used to teach math skills to practice work as skills are learned.

Reith & Evertson, 1988). Each recommendation makes an important contribution, but achievement will be highest when all five are need assistance completing their practice exercises, the teacher may close the lesson.

The literature on direct instruction of students with LD identifies five recommendations in the delivery of instruction that contribute to the effectiveness of that instruction: (a) Obtain frequent active responses from all students, (b) maintain a lively pace of instruction, (c) monitor individual students' attention and accuracy, (d) provide feedback and positive reinforcement for correct responding, and (e) correct errors as they occur (Archer & Isaacson, 1989; Becker & Carnine, 1981; Christenson et al., 1989; Mercer & Miller, 1992; Silbert et al., 1990). Each recommendation makes an important contribution, but achievement will be highest when all five are part of the delivery of instruction (Becker & Carnine, 1981).

Typically, the teacher closes a lesson with three brief steps. First, he or she reviews what was learned during the current lesson, where there may have been difficulty, and where performance may have been particularly good. The review may also include a brief statement of how learning in the current session extended what was already known. Second, the teacher provides a brief preview of the instructional objectives for the next session. Third, she or he assigns independent work. Independent seatwork and homework provide important opportunities for students to apply knowledge and practice skills that they have already learned, thus increasing fluency and retention. Independent practice should, therefore, be carefully selected so that students can actually complete it successfully without assistance from a teacher or parent.

Practice activities are essential components of mathematics instructional programs. Students with LD will generally need more practice and practice that is better designed than students without LD, if they are to achieve adequate levels of fluency and retention. Worksheets are commonly used to provide practice, but the ones that publishers supply are frequently inadequate. Table 2 provides a list of principles for designing and evaluating practice activities for students with LD. For a more detailed discussion of those guidelines, see Carnine (1989).

Model-lead-test presentation procedures have many positive qualities: They are quite easy to learn; they are, with slight variations, generalizable to teaching motor skills, concept discriminations, rule relationships, and strategy tasks (see Archer & Isaacson, 1989; Kameenui & Simmons, 1990; Silbert et al., 1990). By themselves, the presentation procedures have been demonstrated to provide for efficient instruction across age levels, ability levels, and curriculum domains. If students imitate the initial prompts, there is little chance that they will guess and make errors. Because prompts are faded, the chances that students will develop dependence on instructional prompts is reduced. Thus, the students learn to expect to be successful and are less apt to resist instruction than if they are confronted with loosely or ranged instructional procedures where the chances that they will fail are high. Finally, there is empirical evidence in the professional literature that direct instruction procedures have been effectively used to teach math skills to older students with LD (e.g., Perkins & Cullinan, 1985; Rivera & Smith, 1988).

<table>
<thead>
<tr>
<th>Opening</th>
<th>Review pertinent achievements from previous instruction. State the goal of the lesson.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body</td>
<td>Model performance of the skill. Prompt the students to perform the skill along with you. Check the students’ acquisition as they perform the skill independently.</td>
</tr>
<tr>
<td>Close</td>
<td>Review the accomplishments of the lesson. Preview the goals for the next lessons. Assign independent work.</td>
</tr>
</tbody>
</table>
2. Build retention by providing review within a day or two of the initial learning of difficult skills, and by providing supervised practice to prevent students from practicing misconceptions and "misrules."

3. Reduce interference between concepts or applications of rules and strategies by separating practice opportunities until the discriminations between them are learned.

4. Make new learning meaningful by relating practice of subskills to the performance of the whole task, and by relating what the student has learned about mathematical relationships to what the student will learn about mathematical relationships.

5. Reduce processing demands by preteaching component skills of algorithms and strategies, and by teaching easier knowledge and skills before teaching difficult knowledge and skills.


7. Ensure that skills to be practiced can be completed independently with high levels of success.

Note. Adapted from Carnine (1989).

TABLE 3: Options for Presenting and Responding to Math Problems

<table>
<thead>
<tr>
<th>Presentation options</th>
<th>Response options</th>
</tr>
</thead>
</table>
| Construct a problem with actual objects or manipulatives | ✤ Construct a response by manipulating objects  
✤ Choose from an array of possible responses (i.e., multiple choice)  
✤ Make an oral response  
✤ Make a written or symbolic representation |
| Present a problem in a fixed visual display    | ✤ Construct a response by manipulating objects  
✤ Choose from an array of possible responses (i.e., multiple choice)  
✤ Make an oral response  
✤ Make a written or symbolic representation |
| Orally state the problem                       | ✤ Construct a response by manipulating objects  
✤ Choose from an array of possible responses (i.e., multiple choice)  
✤ Make an oral response  
✤ Make a written or symbolic representation |
| Present the problem in written or symbolic form| ✤ Construct a response by manipulating objects  
✤ Choose from an array of possible responses (i.e., multiple choice)  
✤ Make an oral response  
✤ Make a written or symbolic representation |

Note. Adapted from Cawley et al. (1978).

Interactive Teacher Presentations and Student Responses

Outside school, the demand to demonstrate mathematics knowledge and skills emerges in a potentially wide range of representations. Appropriate responses also vary. Cawley, Fitzmaurice, Shaw, Kahn, and Bates (1978) described a model for programming mathematics instruction for secondary students with LD that takes into account both the students’ skills and the various possible representations of mathematics problems. Their instructional program is interactive. Instructional examples are
selected according to (a) the individual student's mastery of specific skills, (b) the mode in which the math problem will be represented, and (c) the mode in which the student will respond to the problem. Problems can be presented in four modes: active constructions or manipulation of objects, fixed visual displays, oral statements, and written or symbolic representations. The student’s response mode may also occur across four options: active constructions or manipulation of objects, identification of a visual display from a set of alternative displays, an oral statement, and a written or symbolic representation.

Table 3 illustrates Cawley et al. (1978) options for presenting and responding to problems. The modes for the teacher's presentation and the student's responses can be mixed; for example, the teacher may present a problem orally, and the student may respond by selecting an appropriate response from alternatives. The 16 different combinations of modes of presentation and response allow instructional programs to be tailored to both the individual needs of students and the realistic representations of mathematical problem-solving situations. Cawley et al. argued that students who have the opportunity to work with varied presentations and response options are more likely to learn generalizable skills than students who spend the greatest proportion of their time making written responses to problems presented in workbook pages.

Peer-Mediated Instruction

Over the past 20 years, educators have taken a growing interest in using students to help with each other's academic achievement. Current initiatives in both general and special education include two major forms of peer-mediated instruction—peer tutoring and cooperative learning. Both enjoy broad support in the empirical literature. Peer tutoring can take various forms. Two classmates may take turns helping each other in one-on-one practice of skills that have been presented earlier. It could also be an arrangement whereby a higher achieving student helps, or monitors the performance of, a lower achieving student (e.g., Maheady, Sacca, & Harper, 1987). Cooperative learning also involves peers assisting each other, but, instead of pairs of students, cooperative learning groups usually comprise three or more students of differing ability levels. Slavin (1983) defined cooperative learning as instructional arrangements in which students spend much of their class time working in small, heterogeneous groups on tasks they are expected to learn and help each other learn. The aspects that contribute to the effectiveness of both approaches will be discussed in this section.

Slavin’s (1983) analysis of 46 studies of cooperative learning resulted in three important conclusions regarding which variables contribute to the success of cooperative learning. First, there was no evidence that group work itself facilitated individual students' achievement. Second, cooperative incentive structures (where two or more individuals depend on each other for a reward that they will share if they are successful) themselves did not have significant effects on the achievement of individual students; group rewards for group products did not improve student achievement. Third, group incentive structures were associated with higher achievement if the performance of individual students was accounted for and was reflected in the group rewards. The simplest method for managing individual accountability is to average individual scores to determine the reward for group members. A second method is to determine group rewards based on whether, or by how much, individual members exceeded their individual criterion. A third method is to assign each student in a group a unique task. Slavin warned, however, that although task specialization provides for individual accountability, it must be accompanied by group rewards. Furthermore, task specialization is not appropriate in situations where the instructional goal is for all members of the group to acquire the same knowledge and skills.

In summary, individual accountability with group rewards contributes to higher levels of achievement than individualized incentive structures, but group work without individual accountability or group rewards does not contribute to higher achievement than might be obtained with individualized task and incentive structures. Slavin (1983) cautioned that peer norms and sanctions probably apply only to individual behaviors that are seen by group members as being important to the success of the group. For example, he suggested that when rewards are group oriented and there is no individual accountability, group norms may apply to the behaviors of only those group members who are considered by the group as most apt to contribute to the quality of the group product.

Several studies of cooperative learning have demonstrated its effectiveness for teaching mathematics skills (e.g., Madden & Slavin, 1983; Slavin, 1984; Slavin & Karweit, 1981; Slavin, Leavely, & Madden, 1984; Slavin, Madden, & Leavely, 1984). Several of those studies (Madden & Slavin, 1983; Slavin, 1984; Slavin & Karweit, 1981) also consistently revealed that low-achieving students in cooperative learning programs enjoyed greater social acceptance by their higher achieving peers and reported higher levels of self-esteem than did low-achieving students in traditional instructional programs. In one particularly relevant study, Slavin and Karweit (1984) compared the effectiveness of cooperative learning and direct instruction for teaching mathematical operations from real-life problems to low-achieving students in ninth grade. They observed higher levels of achievement with cooperative learning than with direct instruction alone. Combining direct instruction with cooperative learning procedures did not produce higher levels of achievement than cooperative learning alone. A caveat, however, is warranted, because a reasonable explanation for the relative superiority of cooperative incentives is that the difficulties the ninth graders in the study experienced were the result of their having forgotten or incompletely learned basic math skills from earlier instruction. In such cases, incentive structures probably play a more important role than the model-lead-test prompting technique. When teaching new concepts or skills, teachers would be well advised to combine direct instruction and cooperative learning.

Strategy Instruction

Students with LD must take an active role in managing their instruction. They must be able to solve problems independently, because teachers and peers are not always available or able to help them. Not only must students master the information and skills taught in their classes, but they must also successfully apply that knowledge and those skills to solve varied and often complex mathematical problems that they encounter outside instruction. Competent students independently select, apply, and monitor strategic procedures to solve complex and novel problems. A common attribute of students with LD, however, is that they do not employ effective learning strategies, unless they are explicitly instructed, and, thus, they are less effective at generalizing skills and knowledge outside the instructional setting (Deshler & Schumaker, 1986; Lenz & Deshler, 1990). If, however, students with learning difficulties are taught to use appropriate learning strategies and are reinforced for using them, they can perform effectively (Deshler & Schumaker, 1986; Ellis, Lenz, & Sabornie, 1987a, 1987b; Lenz & Deshler, 1990; Montague, Bos, & Doucette, 1991; Pressley, Symons, Snyder, & CarigliaBull, 1989).

Deshler and associates (e.g., Deshler & Schumaker, 1986; Lenz & Deshler, 1990; Ellis et al., 1987a, 1987b) have articulated a
model for strategy instruction specifically for secondary students with LD. According to their model, effective strategy instruction follows a process that is consistent with the development of curriculum and instruction that we described in the previous sections of this article. Strategies are selected with reference to the curriculum demands. Teachers manage the instruction of strategies by overtly modeling strategies and then leading students through their applications. Students verbalize their applications of strategies and monitor their own progress. The teacher also provides the students with many opportunities to determine which strategies are appropriate, to use the strategies, and to be rewarded for successful applications.

In the case of mathematics, students confront many quantitative and conceptual relationships, algorithms, and opportunities to apply mathematical knowledge to solve problems. Thus, training for generalization and strategic problem solving can become a ubiquitous part of mathematics curricula. By learning to be active and successful participants in their achievement, students learn to perceive themselves as competent problem solvers. They are more apt to attempt to apply knowledge in novel ways and to persevere to solve difficult problems than if they see themselves as ineffective, likely to fail, and dependent on others to solve novel and difficult problems.

Empirical research on strategy instruction has not been comprehensive in all areas of instruction, but educators should be optimistic and make reasonable attempts to implement strategy instruction. Many studies of strategy instruction have been conducted with secondary students across a variety of academic domains. Evaluations of strategy instruction in mathematics have demonstrated its effectiveness (Carnine, 1980; Case, Harris, & Graham, 1992; Darch, Carnine, & Gersten, 1984; Hutchinson, 1993; Montague, 1992; Montague & Bos, 1986). Because those evaluations have involved only a few areas of math skills, researchers should continue to study the conditions that influence the efficacy of strategy instruction.

### Evaluation of Instruction

It is essential that instructional interventions be evaluated frequently. The academic difficulties of secondary students with LD are diverse and complex. Current research on mathematics instruction for students with learning difficulties is not sufficiently developed to provide teachers with precise prescriptions for improving instruction. Therefore, the best educators’ best efforts will frequently be based on reasonable extrapolations. Unless instructional assessments are conducted frequently and with reference to the students’ performance on specific tasks, it will not be possible to use the information to make rational decisions for improving instruction. To an increasing extent, educators have come to the conclusion that traditional standardized achievement testing does not provide adequate information for solving instructional problems, and that a greater emphasis should be placed on data from functional or curriculum-based measurements (Reschly, 1992).

Assessments of instruction should provide data on individual students’ progress in acquiring, generalizing, and maintaining knowledge and skills set forth in the curriculum. Curriculum-based assessment (CBA) is an approach to evaluating the effectiveness of instruction that has gained substantial attention for its value in the development of effective instructional programs. CBA can be characterized as the practice of taking frequent measure of a student’s observable performance as he or she proceeds through the curriculum. As data are gathered, the measures of student performance are organized, usually graphed, and examined to make judgments of whether the student’s level and rate of achievement are adequate. The teacher’s reflections on the curriculum-based measures of performance and qualitative aspects of the student’s performance may suggest a variety of rational explanations and potentially useful instructional interventions. If the student is making adequate progress, then it is not necessary to modify the program. On the other hand, if progress is inadequate, appropriate interventions might include devoting more time to instruction or practice, engaging the student in higher rates of active responding, slicing back to an easier level of a task, shifting instruction to tasks that are more explicitly or more parsimoniously related to the instructional objective, or changing the incentives for achievement. If curriculum-based measures appear to indicate that the student is having little or no difficulty meeting instructional criteria, the teacher should consider skipping ahead to more difficult tasks.

The value of CBA as a technique for improving quality of instruction can be attributed to the effects it appears to have on teachers’ instructional behavior. First, the collection of valid curriculum-based measures requires that teachers specify their instructional objectives. Efforts to identify critical instructional objectives may also lead teachers to consider how those objectives should be sequenced for instruction. Such considerations can contribute to improvement in the quality of instruction; however, Fuchs and Deno (1991) argued that measurement of subskill mastery is not necessary and that measures of more general curriculum-based measures can provide educators with reliable, valid, and efficient procedures. Second, preparations for implementing CBA are frequently accompanied by specifications of expectations for instruction, such as how many or at what rate objectives will be learned. Fuchs, Fuchs, and Deno (1985) observed that teachers who set clear but ambitious goals for their students tended to obtain higher levels of achievement from their students than teachers who set more modest goals. Third, well-planned efforts to systematically reflect on frequent observations of academic performance are likely to result in more rational assessments of skill achievement than approaches to instruction that do not rely on frequent assessments of skill performance (Fuchs, Fuchs, Hamlett, & Stockler, 1999).

In summary, CBA provides for frequent assessments of student achievement that are directly related to instructional programs. Its use appears to have the effect of more rationally relating instructional decisions to instructional objectives and student difficulties, thus contributing to increased student achievement.

### Educators’ Beliefs About Effective Instruction

There is no disagreement that all students, including those with LD, should be offered the most effective instruction possible. There are, however, diverse opinions among educators about the nature of effective instruction. It would be gratifying if empirical research played a bigger role in directing educational practice, but, frequently, beliefs and convictions play more influential roles. Sometimes educators’ beliefs have positive influences on the development of instruction. For example, if a teacher believes that the source of a student’s failure lies in his or her instructional experience, then the teacher may revise the curriculum, allocate more time for instruction, slice back to a less complex level of the task, or systematically and frequently collect achievement data so that the effects of instructional interventions can be monitored.
Proponents of current efforts to reform mathematics education believe that if the quality of instruction is to be improved, then many educators will have to dramatically change their perspectives on how mathematics should be taught (e.g., National Council of Teachers of Mathematics, 1989). The National Council of Teachers of Mathematics set forth the following goals for all students: (a) to learn to value mathematics, (b) to become confident in their abilities to do mathematics, (c) to become mathematical problem solvers, (d) to learn to communicate mathematically, and (e) to learn to reason mathematically. We believe that these are worthy goals and that in order to reach them, educators must examine their beliefs; however, beliefs that form the basis of the constructivist approach to mathematics education will be insufficient for guiding the development of mathematics curricula, and may be interpreted by educators as legitimizing inadequate instructional practices.

Constructivism is an ideology that is becoming increasingly popular in the current mathematics education reform movement. Its core belief is that knowledge is not transmitted directly from the teacher to the student. Instead, the learner constructs knowledge through active engagement in the process of assimilating information and adjusting existing understandings to accommodate new ways of knowing (e.g., Cobb, 1994a, 1994b; Driver, Asoko, Leach, Mortimer, & Scott, 1994; Gadanidis, 1994; Harris & Graham, 1994; Poplin, 1988a; Pressley et al., 1992). Accordingly, the world that students come to know does not exist as an independent entity outside their minds. Their knowledge of the world is actively changing according to what they already have learned and what they are currently learning.

Descriptions of the constructivist approach depict situations in which (a) teachers possess a considerable knowledge of their subject matter; (b) teachers are able to draw on that knowledge to facilitate student learning under conditions that require a great deal of extemporaneous decision making; (c) students are highly motivated to tackle difficult and challenging tasks in order to develop greater understanding; and (d) it becomes apparent over the course of the lesson that students are acquiring, developing, or constructing more sophisticated understandings than they had before the lesson began. Bereiter (1994) commented that it would be truly exceptional if all or most of these conditions could be met. He identified three substantial obstacles to meeting the conditions: (1) inadequate teacher education, (2) trial-and-error learning, and (3) difficulty with observing progress.

Teacher education is a major obstacle, but, compared to the others, it may be the easiest to overcome. Under relatively unstructured conditions in which students play very active roles in the direction of instruction, it is difficult for even the most knowledgeable teacher to facilitate learning adequately and consistently. The second obstacle is that in constructivist approaches, students often persevere through trial-and-error learning. Under such learning conditions, students with LD are apt to make many more errors than their more capable peers. If they expect to fail, they are prone to give up or to withdraw from instruction.

The last condition (i.e., effecting and acknowledging progress) will be difficult to meet, unless very simple concepts and skills are being taught. Over the course of an instructional session, it must be apparent to both the student and the teacher that progress is being made. Two factors limit the probability that progress will be consistently detected and rewarded: difficulty specifying objectives, and complexity of instructional conditions. The two basic tenets of constructivism—that knowledge cannot be directly transmitted from the teacher to the student, and that students actively construct their own knowledge—suggest that precise, meaningful, and measurable learning objectives may not be established. The CBA approach cited earlier suggests that as precision in measurement of student performance increases, so does student achievement.

In summary, the constructivist perspective, though intuitively appealing, is currently unsupported by empirical research and is logically inadequate for the task of teaching adolescents with LD. It is a set of beliefs that may not be able to be implemented to the satisfaction of those who promote it (see Reid, Kurkjian, & Carruthers, 1994). Unfortunately, it is also a set of beliefs that may fail to encourage the hard work of improving the quality of instruction for students with LD, and may actually be interpreted as justification for haphazard instruction. The premise that secondary students with LD will construct their own knowledge about important mathematical concepts, skills, and relationships, or that in the absence of specific instruction or prompting they will learn how or when to apply what they have learned, is indefensible, illogical, and unsupported by empirical investigations. If instructional objectives are ambiguously organized, the objectives will not be adequately addressed, and students with LD will not achieve them.

Conclusions

Secondary students with LD spend the bulk of their instructional time on very simple math skills. As a result of frequent failure, and of prolonged instruction on such simple skills, it is generally difficult to motivate them to attempt complex tasks or to persist in independent work. By the time that students graduate or drop out of school, they will have made only the most rudimentary achievements. Few will have acquired the levels of application and problem-solving skills necessary to function independently. To a great extent, improvement in mathematics education for secondary students with LD will depend on their receiving better mathematics education while they are in the elementary grades.

We strongly believe that efforts to improve mathematics education must be based on empirical research. There is already a sufficient body of research to serve as the basis for making substantial improvements in educational practice, but much more is needed. Empirical documentation of effective practice will not be sufficient to effect improvements. Compelling research on effective instructional practices is frequently ignored (Carnine, 1992). Instead, appealing but unvalidated trends, such as constructivism and discovery learning, have caught educators' attention. Those ideologies tend to be vague and allow support for haphazard and poorly designed instruction. They are logically antithetical to the existing empirical evidence on best practices for students with LD.

Educational practices that are derived from ideologies must be critically evaluated—and not merely for their fit with the political sensibilities of any particular ideology, but for their effect on the achievement of children and youth.

We are also convinced that teachers must have at hand effective instructional procedures, materials, and other resources. At the present time they must do much of the work of improving mathematics education themselves. Unfortunately, few teachers have sufficient time or training to design and comprehensively evaluate math curricula (Cobb, 1988; Swing, Stoiber, & Peterson, 1988). It is the business of commercial publishers to design instructional procedures and curricula. Instead of attempting to keep up with shifts in ideologies, publishers should attempt to produce empirically defensible tools for teachers.
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