SECONDARY SCHOOL TEACHERS’ CONCEPTIONS ABOUT ALGEBRA TEACHING

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Abstract: In this article secondary school conceptions concerning the purposes of algebra teaching are discussed. The data was gathered by interviews and videotapes. Both newly graduated and experienced teachers were participated in the study. The phenomenographic research method was applied in the investigation. The results indicated that the teachers present algebra as something to do rather than emphasising the central ideas and concepts of algebra. They comprehend algebra as a study of procedures for solving certain kinds of problems in everyday life and problem solving. The key instructions in this conception are simplify and solve.

INTRODUCTION
Teaching of and achievements in mathematics have been criticised in several countries during the last decade. It is generally concluded that school mathematics focuses on developing algorithmic skills rather than mathematical understanding (e.g. Sierpinska 1994; Soro&Pehkonen 1998) and teachers devote much less time and attention on conceptual instead of procedural knowledge (Porter 1989; Menzel and Clarke 1998, 1999). Pupils learn superficially several basic concepts in arithmetic and algebra without understanding (e.g. Hiebert&Carpenter 1992; Sierpinska 1994).

In recent years, there has been a growing interest in mathematics teachers’ conceptions about teaching and learning of mathematics. Teachers’ mathematics-related beliefs and conceptions have been investigated in numerous research reports on the last decade (e.g. Adams and Hsu 1998; Pehkonen 1998). Also student teachers’ conceptions of mathematics teaching have been studied (e.g. Trujillo &Hadfield 1999). There are still few studies concerning mathematics teachers’ beliefs and conceptions of different content areas such as algebra. Most of the earlier studies are dedicated to teachers’ beliefs and conceptions of mathematics, mathematics learning and mathematics teaching (Thompson 1992).

Interest in research about teacher knowledge has also arisen in recent years. Much current work on the development of a qualitative description of teacher knowledge and conceptions has been influenced by Shulman’s model for teacher knowledge (Shulman 1986). Current research on mathematics teachers’ subject matter knowledge, which includes knowledge of the content of a subject area as
well as understanding of the structures of the subject matter (Schulman, 1986, 9), has been investigated in a large number of recent studies (e.g. Tirosh, Fischbein, Graeber & Wilson, 1999; Attorps 2003). The research results are essentially the same: teachers lack conceptual knowledge of many topics in the mathematics curriculum. Recent research on the relationship between teacher knowledge and teaching practice has also pointed out the need to carry out more studies involving specific mathematical topics. Furthermore, the research has shown that the way teachers in mathematics instruct is determined partly by their pedagogical content knowledge i.e. knowledge that is specific to teaching particular subject matter (Schulman 1986, 9). In numerous research reports (e.g. Lloyd 1998) a strong interdependence of conceptions about subject-matter knowledge and pedagogical content knowledge has been documented. It appears that many teachers do not separate their conceptions about a subject specific topic from notions about how to teach that topic. Therefore teachers’ subject matter knowledge influences their planning and classroom decisions (Brophy 1991).

In this study, the phenomenographic approach is used in order to reveal differences between the teachers’ conceptions about algebra teaching. The approach illustrates in qualitatively different ways how a phenomenon is apprehended by individuals (Marton and Booth 1997). A person’s knowledge of the world is regarded as a number of conceptions and relations between them.

In this paper I discuss one of the aspects of teachers’ pedagogical content knowledge. The present study seeks to answer the following question: What pedagogical content conceptions do the secondary school teachers have of the purposes of algebra teaching? The study especially deals with the purposes of teaching the concept of equation.

PEDAGOGICAL CONTENT KNOWLEDGE

Teachers’ knowledge of teaching mathematics is based on their learning experiences in mathematics. This knowledge is developed during the studies of mathematics but most of this knowledge is acquired in teacher education, teacher practice or in the place of work (Ernest 1989, 18). Knowledge that is specifically connected with teaching particular subject matters is called pedagogical content knowledge (Shulman 1986, 9-10; Grossman 1990, 7; cf. Ernest 1989, 17-18). Pedagogical content knowledge is a term to describe the ways of representing and formulating the subject that make it comprehensible to others (Shulman 1986, 9). According to Brown and Borko (1992, 221) one of the most important purposes in teacher education is the acquisition of pedagogical content knowledge. In fact, it is recognised, that this knowledge forms the essential bridge between academic subject matter knowledge and the teaching of subject matter. It includes an understanding of which representations are most appropriate for an idea, which ideas are difficult and easy for learners,
what conceptions and preconceptions that students in different ages hold about an idea. Particularly, if the preconceptions are erroneous conceptions, which they often are, teachers need to improve their knowledge of strategies in order to be successful in reorganising the understanding of learners (Shulman 1986, 9-10). Pedagogical content knowledge also includes conceptions and beliefs about the purposes for teaching a subject at different grade levels (Grossman 1990, 8; Ernest 1989, 20). A teacher in mathematics must have a clear conception of the purpose of teaching specific curricular topics such as algebra at school. According to Picciotto and Wah (1993, 42) the aim of school algebra should be an understanding of the concepts where different mathematical tools and themes are considered to be vehicles and not the purpose of the course itself. Pupils need to absorb concepts such as functions, numbers, variables, operations, equations and mathematical structures; tools and themes may strengthen motivation (cf. The Swedish Board of Education 2000).

The curricula in compulsory schools in Sweden are designed to make clear what all the pupils should learn. ‘Goals to attain’ define the minimum knowledge to be attained by all the pupils by the end of the fifth and ninth year at school (The Swedish Board of Education 2000). According to the curriculum in mathematics, the pupils should for example by the end of the fifth year be able to discover numerical patterns and determine unknown numbers in simple formulae in introductory algebra. Similarly, the pupils should by the end of the ninth year be able to interpret and use simple formulae and solve simple equations. The school in the teaching of mathematics, should aim to ensure that pupils develop their ability to understand and use logical reasoning, draw conclusions and generalise, explain both orally and in writing and provide the arguments for reasoning. The goal of teaching of algebra should be that pupils develop their ability both to understand and to use basic algebraic concepts, expressions, formulae, equations and inequalities (The Swedish Board of Education 2000).

**METHOD**

Ten secondary school teachers in mathematics participated in the study. Five teachers were newly graduated (less than one year’s experience) and five were experienced (between 10 and 32 years’ experience). Data was gathered by interviews and videotapes. The interviews took place in the schools, where the teachers worked, and were recorded. Each interview lasted about two hours. Videotape recordings of six lessons, which the three newly graduated and the three experienced teachers had in algebra, gave further information about their purposes of algebra teaching at the school context. The interview quotations have been marked in the following way: For example I1 = Interview 1, p1 = page 1 in transcribed protocol, V1 = Videotape 1 and M or K 1 - 4 = Person code.
The interpretation of data in the phenomenographic research begins already during the interviews. The respondents’ reactions and feelings how they understand and experience a phenomenon mediate knowledge, which is important to notice as well as what the respondents say. These messages during the interviews facilitate the understanding of the data as a whole. In order to achieve a general picture of the collected data I listened to the tapes, watched videotapes and read transcribed protocols several times. In the transcribed protocols, I found that some conceptions were more frequent than others and details and patterns could be identified in the interviews. I split up the protocols into four categories of description. The categories of description are considered as a main research result in phenomenographic investigations.

“Conception” is the most central concept in phenomenography (Marton and Booth 1997). ‘Conception’ is defined in literature in many different ways. In this investigation Sfards’ definition of ‘conception’ is used, i.e. persons’ subjective conception of an object or a phenomenon (Sfard 1991, 3). Conceptions are regarded as part of teacher knowledge (Grossman 1990).

RESULTS

The teachers proposed different purposes for algebra/equation teaching. Their conceptions have been classified into four qualitatively different categories: (1) Pupils should learn to use equations as a tool in problem solving, (2) Pupils should learn to use equations as a tool in everyday life, (3) Pupils should learn equations in order to express their thoughts from a general point of view and (4) Pupils should learn equations in order to achieve the goals of the mathematics curriculum. The first two purposes of teaching have a practical aspect. They stress a procedure rather than the innermost ideas of equations, which is expressed in the third conception. The fourth purpose has an aspect, which is directly related to curriculum demands in mathematics. A more careful description of the categories now follows.

Conception 1: Pupils should learn to use equations as a tool in problem solving

Pupils should learn to use equations in text problems. (I3, p1, M1)

...they should use equations as a tool in problem solving. (I3, p1, K2)

It is simply as a tool in mathematics. I emphasize that pupils should use equations. (I3, p1 M3)
The conceptions above indicate the practical aspects of equations in problem solving. During a lesson one of the experienced teachers gave the following practical problem. A family consists of four members: two girls Eva and Anna, mum Ulla and dad Kurt. Eva is 10 years. Anna is x years. Ulla is 5 times older than Anna plus 3 years and Kurt is 6 times older than Anna minus 1 year. How old are Anna, Ulla and Kurt if Kurt and Anna are the same age as Eva and Ulla? A teacher was very careful when reading the text problem. She asked the pupils, “What do the words ‘and’ and ‘the same age’ mean?” - Then she wrote an equation together with pupils on the blackboard (V4, K4). One of the pupils asked if it is OK to use ‘ö’ instead of ‘x’. After solving the equation with ‘ö’, the teacher says, “Even if there is no ‘ö’ in your textbooks, you can find a lot of equations there.” (V4, K4)

Some teachers also feel that they cannot realize their ideas and purposes of teaching. One of the experienced teachers complains: “I cannot teach in the way I want, because I must be ‘a police’. Today I cannot implement the things that I really want. I often lose the main thread during lessons.” (I3, p1, M3)

The next section illustrates the purpose of teaching as instructing pupils how to use equations as tool in everyday life.

Conception 2: Pupils should learn to use equations as a tool in everyday life

Pupils should use equations as a tool and they shall see the value of equations in everyday life, for example in connection to calculation of percentages. (I3, p1, M1)

Pupils should see that equations are not only ‘hocus-pocus’ formulas, which can be performed. They should see that equations have to do with reality. (I3, p1, M2)

Pupils get a picture of an equation like Pythagorean theory and see the use of it. They see that x may stand for a side and they get a connection to reality. (I3, p1, K4)

The pupils should get a picture, a real picture, e.g. a telephone bill in which you pay a fixed fee plus a fee for the calls you phone. (I3, p2 M3)

These conceptions stress the practical aspects of equations. They stress a ‘real picture’ aspect of equations in everyday life. The conceptions have a process character. One of the newly graduated teachers says during a lesson in mathematics: “A balance is equality. Equations are also balances. A balance can be an equation.” (V1, K1). An experienced teacher who also uses a balance in order to give a real picture of equations says: “I use a balance in teaching of
equations, sometimes I also draw.” (V4, M4). He gives the following example. We do not know the weight of this stone. We are going to find out the weight by using a balance and different weights (2 hg and 0,2 hg). When the balance was achieved the teacher compared the two sides of the balance with an equation.

“As an equation has two sides like a balance and they are called the left-hand side and the right-hand side and in this case they are equal.....In an equation we have one letter, one unknown. The most usual is x. The weight of the stone is called x.” (V4, M4). The teacher shows that in the left-hand bowl there is a stone and one weight (2hg). In the right-hand bowl there are three weights (2hg+2hg+0,2hg). The bowls are in balance, the equation x + 2 = 2 + 2 + 0,2 can be written. The teacher describes carefully how to solve the equation by using the formal solving procedure.

Examples above illustrate that the teachers try to give a real picture of equations. By using a balance, a telephone bill, applications to calculation of percentages or geometry they try to give to pupils a picture of equations in real context.

The third conception has to do with central ideas in algebra.

Conception 3: Pupils should learn equations in order to express their thoughts from a general point of view

It’s important that the pupils can express their thoughts generally ...(I3, p1, K3)

In this conception ideas like ‘using letters in algebra’ and ‘understanding of algebraic structures’ have a central place. The following example from a mathematics lesson is illustrative. A newly graduated teacher says in a lesson in mathematics: “It is important to learn carefully from the beginning. In this way we can promote knowledge.” (V1, K1). She wonders, if the pupils have worked with equations already at the primary school. She writes on the blackboard:

\[
1 + 1 = 2; 1 + \Box = 5; 1 + \bigtriangledown = 5; 1 + \bigcirc = 5
\]

She asks, “if the pupils recognize this, adding: You have started to use equations already in the first class, but you do not know the very name.” (V1, K1). She points out that different symbols for an unknown factor have the same meaning. She writes on the blackboard \(3 + 4 = 7\). She points to the equals sign and says. “This is called equality.” She writes again, \(2 + 2 = 4\). She asks the pupils: “What is this?” The pupils answer: equality. She continues to write, \(x + 2 = 7\) and she asks: “What do you call this?” The pupils answer: equality. The teacher says: “The equality has another name. It is called an equation, where the left hand-side is equal to the right-hand side.” The teacher gives another example and stresses that the pupils need to learn a method for solving equations in order to
solve more complicated equations. She begins with a simple equation. She writes;

\[ x + 2 = 7 \]

which by using other symbols for 2 and 7 can be written

\[ x + \ast \ast = \ast \ast \ast \ast \ast \ast \]

In order to solve \( x \) you eliminate the two \( \ast \ast \) from both sides

\[ x + \#\# = \#\# \ast \ast \ast \ast \]

\[ x = 5 \]

The teacher stresses: “You should think of a balance when you solve equations. Whatever you do on one side of the equation, you should do the same on the other side.” (V1, K1)

After this she solves the equation above by using the formal method and continues with more complicated examples.

The example above from the lesson in mathematics illustrates that algebra is more than \( x \) and \( y \). The example shows that a letter in algebra can stand for different symbols. The symbols like \( \square \), \( V \) and \( O \) are logically equivalent to \( x \).

Furthermore, the example shows that algebra is more than procedures ‘to solve’, ‘to find out’ or ‘to do something’. It also includes structures. Comparing arithmetical equalities with algebraic structures can be the first step for a learner in transition from arithmetic to algebra.

The fourth conception of the purposes of teaching equations in the compulsory school has to do with the goals in mathematics curriculum. The goals stress that the school in its teaching of algebra should aim to ensure that pupils develop their ability both to understand and to use basic algebraic concepts (The Swedish Board of Education 2000). They also define the minimum knowledge to be attained by all pupils after they have ‘passed’ the fifth and ninth grade at the compulsory school.

Conception 4: Pupils should learn equations in order to achieve the goals in mathematics curriculum

The aim is that all the pupils should achieve the goals related to mathematics curriculum. The aim is to teach so many pupils as possible, so that all at least can get the mark ‘approved’. (I3, p10, M4)

The same teacher points out: “My duty is to teach equations because all the pupils in principle must go to upper secondary school.” (I3, p1, M4)
teacher probably considers the goals in the Swedish syllabuses, which make clear what all pupils should have learnt after they have ‘passed’ the fifth and ninth grade at the compulsory school. At each school and in each class, the teacher must interpret the national syllabuses and together with the pupils plan and evaluate teaching on the basis of the pupil’s preconceptions, experiences and needs.

**Analysis and discussion**

The teachers interviewed have proposed different purposes or aims for the teaching of algebra/equations. According to the Swedish curriculum the teaching of algebra should aim to ensure that pupils develop their knowledge and ability to **understand** and **use** basic algebraic concepts, expressions, formulae, equations and inequalities **as a tool in problem solving** (The Swedish Board of Education 2000). Many of the interviewed teachers stress that the pupils at compulsory school should learn to **use** the concept of equation as a tool in problem solving and in everyday life rather than to **understand** the concept as an abstract entity. The first two purposes of teaching algebra have a practical aspect in the meaning of ‘a tool’, ‘to use’ and ‘a real picture’. They stress a procedure rather that the innermost idea with equations, which is a characteristic of the third conception in the study. In this the aim of algebra teaching is more than the learning of procedures ‘to solve’, ‘to find out’ or ‘to do something’. The third conception includes mathematical structures. Comparing arithmetical equalities with algebraic structures can help a learner to take the first step in transition from arithmetic to algebra. The fourth purpose of algebra teaching has an aspect, which is directly related to curriculum goals in mathematics. These goals define the minimum knowledge to be attained by all pupils in the fifth and ninth year of school. (The Swedish Board of Education 2000). According to the curriculum in mathematics, the pupils should e.g. by the end of the fifth year be able to discover numerical patterns and determine unknown numbers in simple formulae in introductory algebra. Similarly, the pupils should by the end of the ninth year be able to interpret and use simple formulae and solve simple equations (The Swedish Board of Education 2000). Furthermore, the goal of the algebra teaching should be that pupils develop ability both to understand and use basic algebraic concepts.

In my view there are at least two possible explanations why many teachers do not teach mathematics with focus on conceptual understanding. One is that many textbooks do not give enough support for such instruction. Another explanation is that teachers may not have sufficient competence in conceptual knowledge in mathematics. Furthermore teachers may not have had enough opportunities to develop their competence in pedagogical content knowledge. This may be due to deficiencies in school politics, educational research and teacher education. Another explanation could be the lack of understanding of the extensive knowledge that is required to teach any subject matter area. In
educational research there are still few studies, which contribute to the development of teachers’ competence in pedagogical content knowledge in subject-specific area. There is also the following problem: to what extent do the research results from educational studies reach teachers, teacher educators and decision-makers? In teacher education many studies indicate that student teachers do not develop sufficient competence in pedagogical content knowledge in order to teach with focus on conceptual understanding (e. g. Tirosh et al. 1999). Perhaps because many teacher educators lack both their own research experiences and the necessary competence in pedagogical content knowledge and therefore they are not able to give student teachers the opportunity to develop such competence. To develop pedagogical content competence requires both time and resources. However, both of them seem to be an article in short supply in mathematics education.

REFERENCES


Brophy, J. 1991. Advances in research on teaching: Teachers’ knowledge of the subject matter as it relates to their teaching practice. Volume 2. Greenwich, CT:JAI.


